Section 2.4

1. Use the following table to answer question 1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
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<tr>
<td>4</td>
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<td>3</td>
</tr>
</tbody>
</table>

If $H(x) = f(x) \cdot g(x)$, what is $H'(2)$?

24

2. Given $f(x) = 5 \csc x$, find a) $f'(x)$  b) $f''(x)$.

$f'(x) = -5 \csc x \cot x$; $f''(x) = -5(\csc x - 2 \csc^3 x)$

3. The equation of motion for a particle is $s(t) = 5 \cos t + 6 \sin t$, $t \geq 0$, where $s$ is measured in centimeters and $t$ in seconds. Find the velocity function.

$s'(t) = -5 \sin t + 6 \cos t$

4. Find the derivative. $f(x) = x^{10} \cos x$

$f'(x) = 10x^9 \cos x - x^{10} \sin x$

5. Find $f'(x)$ if $f(x) = 4x(\sin x + \cos x)$

$f'(x) = 4(\sin x + \cos x) + 4x(\cos x - \sin x)$

6. An object with weight $P$ is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle $t$ with the plane, then the magnitude of the force is

$F = \frac{cP}{\csc t + \cot t}$, where $c$ is a constant called the coefficient of friction. Let $P = 30$ lb and $c = 0.5$. When (in radians) is the rate of change of $F$ with respect to $t$ equal to zero?

$arctan 0.5$
Section 2.5

7. Find the first derivative of \( y = \tan^4 x \).

\[ y' = 4 \tan^3 x \sec^2 x \]

8. Find the equation of the tangent line for \( y = \cos^3(x) \) at \( x = 0 \).

\[ y = 1. \]

Use the following table to answer question questions # 9 and # 10.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( f'(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
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<td>1</td>
</tr>
</tbody>
</table>

9. If \( h(x) = f(g(x)) \), what is \( h'(1) \)?

6

10. If \( H(x) = g(f(x)) \), what is \( H'(3) \)?

3

11. Find the 28th derivative of \( y = \cos(4x) \).

\[ y^{(28)} = 4^{28} \cos(4x) \]

12. Find the derivative. \( y = (\sec(x))^4 + \cos(x^5) \)

\[ y' = 4(\sec(x))^4 \tan(x) - 5x^4 \sin(x^5) \]

13. Suppose that \( f(x) = \frac{3x}{(2-4x)^4} \). Find the equation of the tangent line of \( f \) at \( x = 1. \)

Round each numerical value to 4 decimal places.

\[ y = -1.3125x + 1.5 \]

14. A Cepheid variable star is a star whose brightness alternately increases and decreases. Suppose that Cephei Joe is a star for which the interval between times of maximum brightness is 5.3 days. Its average brightness is 2.9 and the brightness changes by +/- 0.35. Using this data, we can construct a mathematical model for the brightness of Cephei Joe at time \( t \), where \( t \) is measured in days:

\[ B(t) = 2.9 + 0.35 \sin(2\pi t/5.3) \]

Find the rate of increase after one day. Round each numerical value to 4 decimal places, except \( \pi \). Leave \( \pi \) as \( \pi \).

0.13\pi \cos(0.38\pi)
Section 2.6

15. Find the slope of the tangent line to the curve $5xy^5 + 3xy = 24$ at $(3,1)$ exactly. 
\[ \frac{2}{21} \]

16. For the equation given below, evaluate $y'$ at the point $(2,2)$ to six decimal places.

\[ (4x - y)^4 + 4y^3 = 1328 \]

\[ 4.235294 \]

17. Find the slope of the tangent line to the curve $5\sin x + 4\cos y - 4\sin x\cos y + x = 7\pi$ at $(7\pi, \frac{3\pi}{2})$.

\[ 1 \]

Section 2.7

18. A street light is mounted on a 16 ft tall pole. A 6 ft woman walks away in a straight path from the pole at a speed of 4 ft/sec. How fast is the tip of the woman’s shadow changing when she is 50 ft from the base of the pole?

\[ 6.4 \text{ ft/sec} \]

19. Liquid is pumped into a spherical balloon at a rate of 4 cubic feet per second. How fast is the radius increasing after 2 minutes, in feet per second? The volume of a spherical balloon is $V = \frac{4}{3}\pi r^3$. Round your answer to six decimal places.

\[ 0.013492 \]

20. The radius of a spherical balloon is increasing at a rate of 2 cm per min. How fast is the volume changing when the radius is 12 cm? Round your answer to six decimal places.

\[ 3619.114737 \]
Section 2.8

21. Use a linear approximation to approximate $\sqrt{49.2}$. Write your answer to five decimal places.

$7.01429$

22. Let $y = 4\sqrt{x}$. To five decimal places: Find the change in $y$, $\Delta y$ when $x = 4$ and $\Delta x = 0.2$.

$0.19756$

23. Let $y = 4\sqrt{x}$. To five decimal places: Find the differential $dy$ when $x = 4$ and $dx = 0.2$.

$0.2$

24. Find linear approximation of the function $f(x) = \frac{1}{x}$ and use it to approximate $\frac{1}{1.04}$.

$9.6$

25. The radius of a circular disk is given as 24 cm with a maximal error in measurement of 0.2 cm.

a) Use differentials to estimate the maximum error.

b) What is the relative error?

Round each numerical value to 7 decimal places, except $\pi$. Leave $\pi$ as $\pi$.

$9.6\pi; \; 0.0166667$

Section 3.1

26. Find the exact limit: $\lim_{x \to \infty} \frac{2\sqrt{11}(8)^x + 15,000}{7(8)^x - 9}$.

$\frac{2\sqrt{11}}{7}$

27. Find the exact limit: $\lim_{x \to -\infty} \frac{9}{5^x - 7}$.

$-\frac{9}{7}$

28. The number, $N$, of people who have heard a rumor spread by mass media at time, $t$, is given by $N(t) = a(1-e^{-kt})$.

There are 200000 people in the population who hear the rumor eventually. 5 percent of them heard it on the first day. Find $a$ and $k$, assuming $t$ is measured in days.

$a = 200000; \; k = -\ln(1-5/100)$
Section 3.2

29. For the function \( f(x) = 3x + 6x^{15} \), find the derivative of the inverse function of \( f \) at \( c = -9 \). In other words, find \( (f^{-1})'(c) \) with \( c = -9 \).

\[-1\]

30. Find the exact limit: 
   \( a) \lim_{x \to \infty} [\ln(5 + 3x) - \ln(5 + 2x)] \quad b) \lim_{x \to 0^+} [\ln 5 \sin x] \).
   Round to six decimal places.

\( 0.405465; -\infty \)

Section 3.3

31. Differentiate the function \( f(x) = x^{6x} \).

\[ f'(x) = 6x^{6x} (\ln(x) + 1) \]

32. Let \( f(x) = -16 \ln(\cos x) \). Find the second derivative of \( f(x) \).

\[ f''(x) = 16 \sec^2 x \]

33. Let \( f(x) = \ln[x^6(x + 5)^9(x^3 + 1)^{10}] \). Find \( f'(x) \).

\[ f'(x) = \frac{6}{x} + \frac{9}{x + 5} + \frac{30x^2}{x^3 + 1} \]

Section 3.5

34. Find \( f'(x) \) where \( f(x) = \arcsin^6(2x + 4) \).

\[ f'(x) = \frac{12 \arcsin^5(2x+4)}{\sqrt{1-(2x+4)^2}} \]

35. Let \( f(x) = 2x^2 \tan^{-1}(8x^2) \). Find \( f'(x) \).

\[ f'(x) = 4x \tan^{-1}(8x^2) + \frac{32x^3}{1+64x^4} \]
Section 3.7

36. Use L’Hospital’s Rule to evaluate the limit exactly:
\[ \lim_{x \to 0^+} 4 \sin(x) \ln(x) \]

0

37. Use L’Hospital’s Rule to evaluate the limit exactly:
\[ \lim_{x \to 0} \frac{6^x - 8^x}{x} \]

\[ \ln \left( \frac{3}{4} \right) \]

38. Use L’Hospital’s Rule to evaluate the limit exactly:
\[ \lim_{x \to \infty} \left( 1 + \frac{8}{x} \right)^{\frac{x}{10}} \]

\[ e^{4/5} \]

39. Use L’Hospital’s Rule to evaluate the limit exactly:
\[ \lim_{x \to \frac{\pi}{2}} \left( 7 \cos(-5x) \sec(-7x) \right) \]

-5