1. Prove: Every simple planar graph is 5-list colorable.

2. Recall that $T_{n,r}$ is the complete $r$-partite graph on $n$ vertices such that every part $X$ satisfies $\lfloor \frac{n}{r} \rfloor \leq |X| \leq \lceil \frac{n}{r} \rceil$. Prove: Among all graphs $G = (V,E)$ on $n$ vertices with $\omega(G) \leq r$, the one with the most edges is $T_{n,r}$.

3. Recall that a graph is transitive if for every pair of vertices $x,y$ there is an automorphism of $G$ that sends $x$ to $y$. Prove: Every connected, transitive graph with an even number of vertices has a perfect matching. (Be sure you use all three hypotheses.)

4. Let $k, n \in \mathbb{N}$, and suppose $G$ is an $A,B$-bipartite with $|A| = n = |B|$ such that $\delta(G) \geq k$ and for all $X \subseteq A,Y \subseteq B$, if $|X|,|Y| \geq k$ then $|E(X,Y)| \neq \emptyset$. Prove: $G$ has a perfect matching.

5. Recall that an $x,X$-fan is a set of $|X|$ internally disjoint $x,X$-paths whose ends in $X$ are distinct. Let $G = (V,E)$ is a graph with $x \in V$ and $Y,Z \subseteq V$ and $k = |Y| = |Z| - 1$. Suppose $Q = \{ Q_y : y \in Y \}$ is an $x,Y$-fan in $G$, where each $Q_y$ is an $x,y$-path. Similarly, suppose $R = \{ R_z : z \in Z \}$ is an $x,Z$-fan in $G$, where each $R_z$ is an $x,z$-path. Prove: There exists an $x, (Y + z)$-fan in $G$ for some $z \in Z$. [Hint: Add a new vertex $w$ whose neighborhood is $Z$ and apply a theorem.]