Name____________________ Student ID No. ____________________

Directions:

- There are six multiple choice questions, of which you must answer five, each worth 6 points. You **must** indicate which problems you want to be graded. The default will be that problems 1-5 will be graded. There are five true false questions worth 4 points each, and three free response questions worth a total of 50 points.
- You must show your work on all questions, **including multiple choice.**
- For the true/false questions, you must give a clear and correct explanation to justify your answer. If the answer is false, you may use a counter example.
- **If you do not show your work, you will receive zero credit for that problem.**
- Partial credit is only available on the free response problems.
- Read all the questions carefully.
- Put the final answer to the space provided in an orderly fashion. Box your final answers. No partial credit will be given if more than one answer is given, or if it unclear which answer is meant to be your final answer.

Honor Statement

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and your instructor. Furthermore, you agree not to discuss this exam with anyone in any section of MAT 272 until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any exam proctor.

Signature:

Multiple choice problems to be graded (choose 5 out of 6): ____________________

Multiple Choice

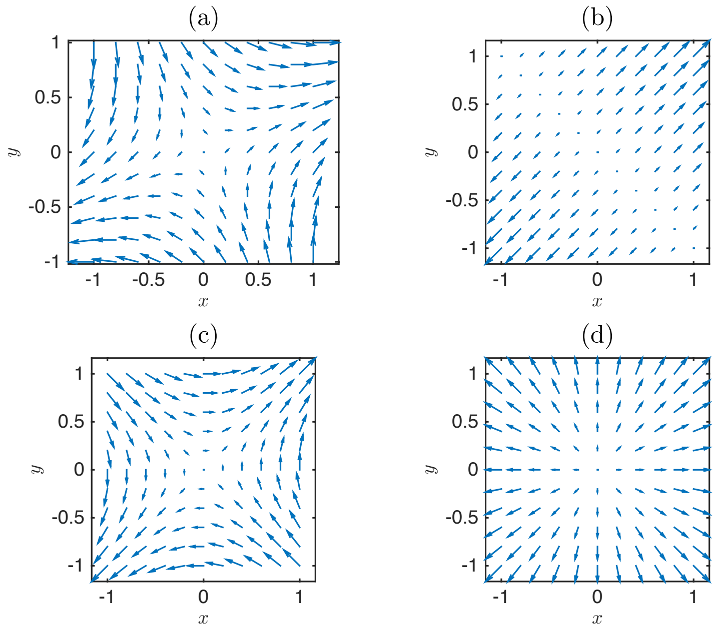
1. Which of the following are conservative vector fields?

   (a) $< e^x \cos y + yz, xz - e^x \sin y, xy + z >$
   (b) $< yz, xz, xy >$
   (c) $< xz, yz, xy >$
   (d) (a) and (b)
   (e) (a) and (c)
   (f) (b) and (c)
   (g) (a), (b), and (c)

2. If a parametric surface given by $\mathbf{r}_1(u,v) = f(u,v)\mathbf{i} + g(u,v)\mathbf{j} + h(u,v)\mathbf{k}$, with $-3 \leq u \leq 3, -5 \leq v \leq 5$ has surface area equal to 4, the surface area of the parametric surface given by $\mathbf{r}_2(u,v) = 4\mathbf{r}_1(u,v)$, with $-3 \leq u \leq 3, -5 \leq v \leq 5$ is

   (a) 16
   (b) $\frac{1}{4}$
   (c) 8
   (d) 64
   (e) Not enough information or none of the above.

3. Which of the following is a graphical representation of $\mathbf{F} =< x+y, x+y >$?



(e) Not enough information or none of the above

4. The work done by the force field $\mathbf{F} =< 3x, 3y, 3 >$ on a particle that moves along the helix $\mathbf{r}(t) =< 5\cos t, 5\sin t, 7t >$ from $(5, 0, 0)$ to $(5, 0, 14\pi)$ is

(a) 150π

(b) $150\pi + 42\pi^2$

(c) 42π

(d) 6π

(e) Not enough information or none of the above

5. Assume $\mathbf{F}$ is a sufficiently differentiable vector field and ϕ is a sufficiently differentiable scalar-valued function. Consider the following expressions:

   - (i) $\nabla(\nabla \cdot \phi)$
   - (ii) $\nabla \times \nabla\phi$
   - (iii) $\nabla \times (\nabla \cdot \mathbf{F})$
   - (iv) $\nabla\mathbf{F}$
   - (v) $\nabla \times (\nabla \times \mathbf{F})$

   Which of the above expressions make sense?

   (a) (i) and (v)
   (b) (ii) and (v)
   (c) (ii), (iv), and (v)
   (d) (i), (iii), and (v)
   (e) all of the above

6. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle xz, yz, xy \rangle$, and C is the circle $x^2 + y^2 = 4$ in the xy plane with counterclockwise orientation. (Hint: use Stoke's theorem)

   (a) π
   (b) 0
   (c) 1
   (d) 2π
   (e) Not enough information or none of the above

True/False: Make sure to **justify** your answer!

1. The vector field $\mathbf{F} =< f(x), g(y) >$ is conservative on $\mathbb{R}^2$.

2. The vector field $\mathbf{F} =< y, x >$ has both zero circulation along and zero flux across the unit circle centered at the origin.

3. The vector $\frac{\partial}{\partial u}\mathbf{r} \times \frac{\partial}{\partial v}\mathbf{r}$ is tangent to the surface parameterized by $\mathbf{r}(u, v)$.

4. Two vector fields in $\mathbb{R}^3$ with the same divergence differ by a constant vector field.

5. If $\mathbf{F} =< x, y, z >$ and S encloseds a region D, then $\int\int_S \mathbf{F} \cdot \mathbf{n} dS$ is three times the volume of D.

Free Response

1. Let $\mathbf{F} =< z^2x, \frac{1}{3}y^3 + \tan z, x^2z + y^2 >$ and S be the top half of the sphere $x^2 + y^2 + z^2 = 1$. Note that S is not a closed surface.

   (a) (8 points) Let S_1 be the disk $x^2 + y^2 \leq 1$ oriented downward, and define $S_2 = S \cup S_1$. Calculate $\int\int_{S_2} \mathbf{F} \cdot \mathbf{n} dS$. Hint: Use the Divergence Theorem.

   (b) (8 points) Now calculate $\int\int_{S_1} \mathbf{F} \cdot \mathbf{n} dS$.

   (c) (4 points) Use parts (a) and (b) to evaluate $\int\int_{S} \mathbf{F} \cdot \mathbf{n} dS$.

2. Consider the vector field $\mathbf{F} =< x - y, x >$ in $\mathbb{R}^2$.

(a) (6 points) Make a sketch of $\mathbf{F}$.

(b) (2 points) Is $\mathbf{F}$ a conservative vector field?

(c) (6 points) Calculate the circulation around the unit circle with counterclockwise orientation.

(d) (6 points) Calculate the flux of $\mathbf{F}$ across the unit circle with counterclockwise orientation in the xy plane.

3. (10 points) Find the flux of $\mathbf{F} =< x, y, z >$ through the curved surface of the cylinder $x^2 + y^2 = 4$ bounded below by the plane $x + y + z = 2$ and above by the plane $x + y + z = 4$ and oriented away from the z-axis. The surface is displayed below from three perspectives (angles of view).