MEMORANDUM

DATE: April 2, 2021

TO: Faculty and Students

FROM: Professor(s) Sebastien Motsch
Chair/Co-Chairs of Dylan Weber
Defense for the PhD in Applied Mathematics
Committee Members Hans Dieter Armbruster
John Fricks
Nicolas Lanchier
Rodrigo Platte

DEFENSE ANNOUNCEMENT
Candidate: Dylan Weber
Defense Date: 04/14/2021
Defense Time: 3:00 PM
Virtual Meeting Link: https://asu.zoom.us/j/81980986827
Title: Network based models of opinion formation: consensus and beyond

Please share this information with colleagues and other students, especially those studying in similar fields. Faculty and students are encouraged to attend. The defending candidate will give a 40 minute talk, after which the committee members will ask questions. There may be time for questions from those in attendance. But, guests are primarily invited to attend as observers and will be excused when the committee begins its deliberations or if the committee wishes to question the candidate privately.

ABSTRACT
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How are opinions formed? What mechanisms cause individuals to think similarly or in disparate ways? Understanding the evolution of opinions is a delicate task as the dynamics of how one changes their opinion based on their interactions with others are unclear. The rise of social media has drastically changed our conception of opinion formation. Rather than a few central outlets acting as a common and external frame of reference, there are now a myriad of smaller actors that steer opinions one way or the other. Therefore, opinion formation can be argued to be an example of self-organized dynamics where large scale behaviors emerge without a central authority similar to a flock of birds or a shoal of fish. In this dissertation we present several contributions to this body of work.

First, we study two linear models of opinion formation - one stochastic in nature and one deterministic. Both models are defined in terms of an underlying graph; we study how the structure of the graph affects the long time behavior of the models in all possible cases of graph topology. We are especially interested in the emergence of a consensus among the agents and provide a condition on the graph that is necessary and sufficient for convergence to a consensus in both models. This investigation reveals several contrasts between the models - notably the convergence rates - which are explored through analytical arguments and several numerical experiments.

Next, we propose and study a variant of the well known bounded confidence dynamics with the goal of inducing unconditional convergence to a consensus. The defining feature of these dynamics which we name the No one left behind dynamics is the introduction of a local control on the agents which preserves the connectivity of the interaction network. We rigorously demonstrate that these dynamics result in unconditional convergence to a consensus. The qualitative nature of our argument prevents us quantifying how fast a consensus emerges, however we present numerical evidence that sharp convergence rates
would be challenging to obtain for such dynamics. Finally, we propose a relaxed version of the control. The dynamics that result maintain many of the qualitative features of the bounded confidence dynamics yet ultimately still converge to a consensus as the control still maintains connectivity of the interaction network.

In a slight departure from opinion formation we study a model hierarchy inspired by the food seeking behavior of a slime mold. Starting from a particle model who’s interaction term shares many features with the agent based models of opinion formation, we derive a macroscopic aggregation-diffusion equation for the evolution of slime mold under the assumption of propagation of chaos in the large particle limit. We analyze properties of the macroscopic model in the stationary case and study the behavior of the slime mold between food sources. The efficient numerical simulation of the aggregation-diffusion equation allows for a detailed analysis of the interplay between the drift, interaction and diffusion regimes.

Finally we introduce a model aimed at a better understanding of the polarization of opinions in addition to consensus. While the use social media has increased connectivity among individuals in a sense, opinions have become more polarized however most models of opinion formation only include attractive forces among the agents and usually only capture polarization through an echo-chamber effect similar to the bounded confidence dynamics. The addition of repulsive forces while natural from a modeling perspective is mathematically challenging as repulsion can cause divergence of the model in question. We introduce a model which we dub the Döppelganger model which includes repulsive forces. We show that this model remains bounded and characterize its convergence in the case where the interaction network is static.