1. In how many ways can \( n \) objects be distributed in \( m \) boxes if

(a) objects are identical, boxes are distinct, and every box is non-empty;

(b) objects are distinct and boxes are identical;

(c) objects are distinct, boxes are distinct, and exactly two boxes are empty;

(d) objects are identical, boxes are distinct, and every box has at most \( k \) objects in it.

2. Show that any sequence of \( n^2 + 1 \) real numbers contains either an increasing sub-sequence of length \( n + 1 \) or a decreasing sub-sequence of length \( n + 1 \).

3. Let \( B_n \) denote the \( n \)th Bell number.

(a) Show that \( B_n = \sum_{i=1}^{n} \binom{n-1}{i-1} B_{n-i} \) for every \( n \geq 1 \).

(b) Find the exponential generating function for \( B_n \).

4. (a) Find the number of \( k \)-element subsets of \( \{2, \ldots, 2n\} \) that contain no consecutive integers.

(b) Consider colorings of \( \{1, \ldots, 2n\} \) with red and blue with the property that if \( i \) is red then \( i-1 \) is red to show:

\[
\sum_{k=0}^{n} (-1)^k \binom{2n-k}{k} 2^{2n-2k} = 2n + 1.
\]

5. The edges of the following figure (obtained from a regular hexagon) are colored with three possible colors, Red, Blue, and Green. Two colorings are considered identical if the figure can be rotated or flipped to obtain one from another.

(a) Find the cycle index of the group of symmetries acting on the edges of the figure.

(b) Find the number of non-identical colorings in which color Red appears at most once.