Quantum Computation.
Instructor: T.Taylor
Time: 3:00 – 4:15 TuTh
Location: ASU Sync Schedule
Class #: 25077 (APM 598)

Textbook : Quantum Computation, A Gentle Introduction, by Rieffle and Polak.

Description: This course primarily covers the theory of quantum computation, although there will be an experimental demonstration or two thrown in. Much of the focus will be on elucidating some of the mathematical mechanics of the situation, because this is much of the reason that physics of subject is difficult to grasp. Our intention is to cover the textbook from the beginning to the discussion of quantum error correcting codes.

Prerequisites:

A course in Linear Algebra is a prerequisite. Anundergraduate physics course that covers quantum mechanics would be desirable but is not necessary.

Week 1:

The nature of reality

- Particles and Waves
- Experiments:
  - light,
  - lasers,
  - flashlights
  - polarization
- Linear Polarization
- Circular Polarization
- Polarization and vectors

Week 2: (Exercises 2.1, 2.3,2.5, 2.6, 2.8,2.12)

Reality and Math for a Single Particle

- Complex numbers and complex vectors
- Superposition
- Vectors say something but they say too much
- Greatness is overrated. So is phase.
- Restrictions and quotient spaces
- Spheres are perfect
  - not the sphere you thought, though
  - cinching the belt much too tightly (having difficulty breathing?)
  - the Bloch sphere
- Superposition, again
  - with quotients.
- Measurements,
  - projections,
  - state collapse
  - probabilities
- A Quantum Key exchange

Week 3 (Exercises 3.2, 3.3, 3.8, 3.10, 3.14, 3.15)
Reality and Math for Multiple (Two) Particles
- New Vector Spaces from Old
- Cartesian products and direct sums
- The tensor product, with too much love.
  - bilinearity
  - universality
  - basis independence
  - order matters. Here's how (non-commutativity)
- Tensor geometry, demystified more than you'd like
  - decomposibility and entanglement
  - bilinearity is not linearity
  - the decomposable quadric
- Pauli Matricies
- Measurement

Week 4 (Exercises 4.2, 4.5, 4.7, 4.10, 4.16, 4.19, 4.20 plus handout)
Tensor Algebra, as if you hadn't already heard too much.
- Multilinearity
- Order matters more
- Linear Transformations, in part and in whole
- The tensor product vs any old bilinear map
- Basis free, but factors matter

Measurement and Computation
- Projections
- Probability
- Mapping unit vectors to probability
- Hermitian operators and measurement
  - eigenvalues
  - eigenvectors
  - orthonormality
  - what these have to do with projections

Week 5 (Exercises 5.2, 5.5, 5.8, 5.11, 5.12, 5.17)
Exam 1 (Chapters 1-4)
Quantum State Transformations as a Model for Computation
- Unitary Transformations
  - no cloning, because why would it?
- Quantum Gates
  - Pauli
  - Hadamard

    - Tensor products of single bit operators
    - Controlled NOT and other controlled gates
- Applications
    - Dense Coding
    - Teleportation
- Unitary transformations as circuits. Circuits as unitary transformations.

Week 6 (Exercises 6.2, 6.3, 6.5)
Quantum Computation doing Classical Computations
- Unitarity and reversibility
- Reversibility for classical computations
- Quantum circuits for arithmetic operations
    - and
    - addition
    - modular addition
    - modular multiplication
    - modular exponentiation

Week 7-8 (Exercises 7.2, 7.3, 7.5, 7.6, 7.7)
Quantum Algorithms
- Useful things to do with superposition and parallelism
- Complexity of quantum algorithms
- Examples of quantum algorithms
- Quantum subroutines
    - disentangling qbits
    - phase change for a subset of basis vectors
    - state dependent phase shift
    - state dependent single qbit amplitude shifts
- Classical quantum computations
- Models and Complexity Classes
- Quantum Fourier Transform

Week 9 (Exercises 8.2, 8.3, 8.5, 8.6, 9.2, 9.4, 9.7,9.10)
Shor's Algorithm
- Reduction to finding the period.
- Factoring
    - quantum core,
    - getting the period from the measurement
- Efficiency
- Generalizations
    - discrete logarithm
    - hidden subgroups

Grover's Algorithm
- Quantum Search
- Amplitude amplification

- Optimality
- Derandomization

Week 10 (Exercises 10.2, 10.4, 10.6)
Test 2 (State transformation, unitarity, reversibility, quantum arithmetic, quantum complexity, Shor and Grover algorithms)

Quantum Subsystems
- Quantum subsystems and mixed states
  - density operators
  - the geometry of mixed states
  - Von Neuman entropy

Week 11 (Exercises 10.10, 10.11, 10.12, 10.13, 10.16, 10.17, 10.24, 10.25)
Quantum Subsystems and Entanglement
- Classifying Entangled states
  - bipartite quantum systems
  - classifying bipartite pure states
  - quantifying entanglement in bipartite states
- Density operator formalism of measurement
- Transformations of quantum subsystems and decoherence
  - superoperators and sum decompositions
  - relation between state transformations and measurements
  - decoherence

Quantum Codes
- Examples of quantum error correction
  - correcting single qbit flips
  - correcting single qbit phase flips
  - correcting all single qbit errors

Week 12-13 (Exercises 11.1, 11.2, 11.6, 11.9, 11.11, 11.17, 11.18)
Quantum Error Correction
- Framework for Error Correction
  - classical error correcting codes
  - quantum error correcting codes
  - correctable sets of errors for classical codes vs quantum codes
  - correcting errors using classical codes vs quantum codes
  - computing on encoded quantum states
  - superpositions and mixtures of correctable errors are correctable
  - classical and quantum independent error models
- CSS Codes
  - dual classical codes
  - construction of css codes from classical codes
  - Steane code
- Stabilizer Codes

    - binary observables for quantum error correction
    - Pauli observables
    - computing on encoded stabilizer states
- CSS codes as stabilizer codes

Week 14
Review

Final Exam: comprehensive