MAT 271

Exam 1

SAMPLE

NOTE: This is only a sample to give an idea of form and length it is not meant to indicate exact content of your exam. The actual material covered on each exam varies from semester to semester.

1. (10 points) True/False, circle your choice. In this problem, only the answer will be graded; you do **not** need to show any work!

   (a) $\frac{1}{3}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx = \tan^{-1}\left(\frac{x}{3}\right)+C.$

   True **False**

   (b) $\int_1^\infty \frac{1}{x}\,dx$ converges.

   True **False**

   (c) $\int_{-2}^{2} \frac{x^2\sin x}{1+x^2}\,dx = 0.$

   True **False**

   (d) $\int_0^{\pi/2} \sin(t^2)dt = \int_0^{\pi/2} \frac{\sin(u)}{2\sqrt{u}}\,du.$

   True **False**

   (e) The form for the partial fraction decomposition of $\frac{1}{(x^2+x+1)^2(x-3)^2}$ is $\frac{A}{x-3}+\frac{B}{(x-3)^2}+\frac{Cx+D}{(x^2+x+1)^2}.$

   True **False**

2. (20 points) The parts of this problem are independent. In this problem, only the answer will be graded; you do **not** need to show any work!

   a) Some values of a function f are given in the table below. Use Simpson's Rule with $n = 4$ to approximate $\int_1^9 f(x)\,dx$.

x	1	3	5	7	9
$f(x)$	3	4	5	1	2

   b) What trigonometric substitution would you use for

$$\int \frac{1}{x^2\sqrt{9-x^2}}\,dx?$$

   c) Find $\displaystyle\int \frac{dx}{2x^2+4x+4}$.

   d) Find $\displaystyle\int \tan^3 x \sec x\,dx$.

   e) Find $\displaystyle\int \cos^2 x\,dx$.

3. (10 points) Find the indefinite integral

$$\int x^2 e^{3x}\, dx.$$

4. (10 points) Evaluate the improper integral or show that it diverges:

$$\int_0^\infty e^{-5x} dx.$$

Solutions

1. (10 points) True/False, circle your choice. In this problem, only the answer will be graded; you do **not** need to show any work!

   (a) $\frac{1}{3}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx = \tan^{-1}\left(\frac{x}{3}\right)+C.$

   [True] **False**

   (b) $\int_1^{\infty} \frac{1}{x}\,dx$ converges.

   True **[False]**

   (c) $\int_{-2}^{2} \frac{x^2\sin x}{1+x^2}\,dx = 0.$

   [True] **False**

   (d) $\int_0^{\pi/2} \sin(t^2)dt = \int_0^{\pi/2} \frac{\sin(u)}{2\sqrt{u}}\,du.$

   True **[False]**

   (e) The form for the partial fraction decomposition of $\frac{1}{(x^2+x+1)^2(x-3)^2}$ is $\frac{A}{x-3}+\frac{B}{(x-3)^2}+\frac{Cx+D}{(x^2+x+1)^2}.$

   True **[False]**

2. (20 points) The parts of this problem are independent. In this problem, only the answer will be graded; you do **not** need to show any work!

   a) Some values of a function f are given in the table below. Use Simpson's Rule with $n = 4$ to approximate $\int_1^9 f(x)\,dx$.

x	1	3	5	7	9
$f(x)$	3	4	5	1	2

$$\boxed{\frac{70}{3}}$$

   b) What trigonometric substitution would you use for

$$\int \frac{1}{x^2\sqrt{9-x^2}}\,dx?$$

$$\boxed{x = 3\sin\theta}$$

   c) Find $\displaystyle\int \frac{dx}{2x^2+4x+4}$.

$$\boxed{\frac{1}{2}\tan^{-1}(x+1)+C}$$

   d) Find $\displaystyle\int \tan^3 x \sec x\,dx$.

$$\boxed{\frac{1}{3}\sec^3 x - \sec x + C}$$

   e) Find $\displaystyle\int \cos^2 x\,dx$.

$$\boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}$$

3. (10 points) Find the indefinite integral

$$\int x^2 e^{3x}\, dx.$$

Solution:

$$u = x^2 \qquad dv = e^{3x}dx$$
$$du = 2x dx \qquad v = \frac{1}{3}e^{3x}$$

Setting $I = \displaystyle\int x^2 e^{3x}\, dx$, we get

$$I = \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\int x e^{3x}\, dx.$$

$$u = x \qquad dv = e^{3x}dx$$
$$du = dx \qquad v = \frac{1}{3}e^{3x}$$

$$\begin{aligned} I &= \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\left[\frac{1}{3}x e^{3x} - \frac{1}{3}\int e^{3x} dx\right] \\ &= \frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{9}\left(\frac{1}{3}e^{3x}\right) + C \\ &= \boxed{\frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + C} \end{aligned}$$

4. (10 points) Evaluate the improper integral or show that it diverges:

$$\int_0^\infty e^{-5x}dx.$$

Solution:

$$\begin{aligned}\lim_{b\to\infty}\int_0^b e^{-5x}dx &= \lim_{b\to\infty}\left(-\frac{1}{5}e^{-5x}\Big|_0^b\right)\\ &= \lim_{b\to\infty}\left(\frac{1}{5}-\frac{1}{5}e^{-5b}\right) \quad (e^{-5b}\to 0)\\ &= \frac{1}{5}-0\\ &= \frac{1}{5}\end{aligned}$$

Hence,

$$\boxed{\int_0^\infty e^{-5x}dx = \frac{1}{5}.}$$

Scratch

2. Work.

a)

$$S_4 = \frac{\Delta x}{3}[f(1) + 2f(3) + 2f(5) + 2f(7) + f(9)], \quad \Delta x = \frac{9-1}{4} = 2$$
$$= \frac{2}{3}[3 + 4*4 + 2*5 + 4*1 + 2] = \frac{2}{3}(3 + 16 + 10 + 4 + 2) = \frac{70}{3}$$

b) there is no work

c)

$$\int \frac{dx}{2x^2 + 4x + 4} = \frac{1}{2}\int \frac{dx}{x^2 + 2x + 2} = \frac{1}{2}\int \frac{dx}{x^2 + 2x + 1 + 1}$$
$$= \frac{1}{2}\int \frac{dx}{(x+1)^2 + 1} = \frac{1}{2}\tan^{-1}(x+1) + C$$

d)

$$\int \tan^3 x \sec x \, dx = \int \tan^2 x \tan x \sec x \, dx = \int (sec^2 x - 1)x \tan x \sec x \, dx$$

Set $u = \sec x$, then $du = \tan x \sec x \, dx$ and the integral becomes

$$\int (u^2 - 1) du = \frac{1}{3}u^3 - u + C = \frac{1}{3}\sec^3 x - \sec x + C.$$

e)

$$\int \cos^2 x \, dx = \int \frac{1 + \cos 2x}{2} dx = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$