# TEST 1 REVIEW

## 10.1 : 3-D Coordinate System

1. Find the distance from (2,-5,3) to each of the following;

   (a) the *xy*-plane
   (b) the *yz*-plane
   (c) the *xz*-plane
   (d) the *x*-axis
   (e) the *y*-axis
   (f) the *z*-axis

2. Find center and radius of the sphere:

   $$2x^2 + 2y^2 + 2z^2 = 4x - 20z + 1$$

3. What are the projections of the point (2,1,3) on the coordinate planes? (That is, the xy-plane, the yz- plane and the xz-plane.)

4. Find the equation of the sphere that passes through the origin and whose center is (1,2,3) .

5. Which of the following points is closest to the xy-plane? A(2,1,3), B(2,5,8) or C(2,4,-1)

6. Write down an inequality which describes the solid ball of radius 4 centered at (1,4,7).

## 10.2 : Vectors

1. A woman walks due west on the deck of a ship at 4 mi/h. The ship is moving north at a speed of 20 mi/h. Find the speed of the woman relative to the surface of the water. Round the result to the nearest tenth..

2. A horizontal clothesline is tied between two poles, 12 meters apart. When a mass of 4 kilograms is tied to the middle of the clothesline it sags a distance of 2 meters. What is the magnitude of the tension on the ends of the clothesline?

3. (a) Find a unit vector in the same direction as $\langle 1,2,5 \rangle$
   (b) Find a vector of length 5 in the same direction as $\langle 1,2,5 \rangle$
   (c) Find a vector of length 3 in the opposite direction as $\langle 1,2,5 \rangle$

4. Let $\boldsymbol{a} = \langle 0,3,-1 \rangle$ and $\boldsymbol{b} = \langle 4,-3,-1 \rangle$. Compute
   (a) $\boldsymbol{a} + \boldsymbol{b}$ (b) $3\boldsymbol{a} + 4\boldsymbol{b}$ (c) $|\boldsymbol{a}|$

5. What is the terminal point of the vector $\boldsymbol{a} = < 1,2 >$ based at $P = (5,5)$ ?

6. Let $\boldsymbol{a} = < 1,2 >$ and $\boldsymbol{b} =< 4,-3 >$ Show that there are scalars $s$ and $t$ so that $s\boldsymbol{a} + t\boldsymbol{b} =< 5,-12 >$

**10.3 : Dot Product**

1. Find the values of $x$ such that the vectors $< 3,2,x >$ and $< 2x,4,x >$ are orthogonal
2. Find the angle between the vectors, if

   **a** = <6, 0> and **b** = <6, 6>
3. Find the scalar projection of **b** onto **a**:

   **a** = <4,2> and **b** = <9,10>
4. Find the vector projection of **b** onto **a**:

   **a** = <4,2> and **b** = <9,10>
5. A force $\boldsymbol{F} = 6\boldsymbol{i} - 4\boldsymbol{j} + 5\mathbf{k}$ that moves an object from the point (0,8, 6) to the point (4, 14, 24) along a straight line. The distance is measured in meters and the force in Newtons.
   (a) Find the work done by the force.
   (b) Find the component of the force parallel to the displacement vector.
   (c) Find the component of the force orthogonal to the displacement vector.
6. Assume that $u \cdot v = 8, \ = \|u\| = 9, \ \ \|v\| = 6.$ What is the value of $8u \cdot (8u - 6v)$?

**10.4 : Cross Product/Scalar Triple Product**

1. Find a unit vector that is orthogonal to both $9\mathbf{i} + 9\mathbf{j}$ and $9\mathbf{i} + 9\mathbf{k}$.
2. Let $\mathbf{v} = 7\,\mathbf{j}$ and let **u** be a vector with length 5 that starts at the origin and rotates in the $xy$ - plane. Find the maximum value of the length of the vector $|\mathbf{u} \times \mathbf{v}|$.
3. Find the volume of the parallelepiped with adjacent edges $PQ$, $PR$, and $PS$:
   $P$ ( 1, 2, 3 ), $Q$ ( 3, 5, 4 ), $R$ ( 3, 2, 5 ), $S$ ( 4, 2, 3 )
4. Find the area of the triangle with vertices (3,4,-1), (2,5,4), (1,6,-2)
5. You are looking down on a map. A vector $\boldsymbol{u}$ with $|\boldsymbol{u}| = 3$ points north and a vector $\boldsymbol{v}$ with $|\boldsymbol{v}| = 4$ points northeast. In which direction does the crossproduct $\boldsymbol{u} \times \boldsymbol{v}$ point?
   (A) south (B) northwest (C) up (D) down

**10.5 : Equations of Lines & Planes**

1. Find the distance between the planes $5x-2y+z-1=0$, $5x-2y+z+4=0$
2. Find an equation of the plane through $(1,1,-2)$, $(-3,-4,2)$ and $(-3,4,1)$.
3. Find the equation of the line through $(2,-2,4)$ and perpendicular to the plane $-x + 2y + 5z = 12$.
4. Find parametric equations for the line through $(-2, 1, 1)$ and $(2, 3, 5)$.
5. Find an equation of the plane that passes through the point $(4, 0, -2)$ and contains the line

   $x = 10 - 3t, y = 10 + 8t, z = 6 + 7t$

6. Find an equation of the plane that passes through the line of intersection of the planes $x - z = 2$ and $y + 3z = 7$ and is perpendicular to the plane $5x + 3y - 2z = 8$.

7. Find a parametric equation for the line through the point (–6, 9, 3) and parallel to the vector <7, 3, -7>

8. Find the point of intersection between the lines

$$L_1: \frac{x-17}{3} = \frac{y-58}{8} = \frac{z-23}{2}$$

and

$$L_2: \frac{x-49}{7} = \frac{y-26}{4} = z - 15$$

9. Find an equation of the plane with $x$-intercept = 9, $y$-intercept = 1, and $z$-intercept = -3.

10. Find the point at which the line $x = 5 - t, y = 4 + t, z = 5t$ intersects the plane $x - y + 4z = 19$.

11. Find the angle between the planes $-5x + 4y + 3z = -2$ and $-x - 3y + 3z = 5$.

12. Find the distance between the plane $2x + y - 3z + 5 = 0$ and the origin.

**10.6 : Cylinders & Quadric Surfaces**

1.
   a. Find an equation of the sphere that passes through the point (6, −2, 3) and has center (−1, 2, 1).
   b. Find the curve in which this sphere intersects the $xy$-plane

2. Write an inequality to describe the half-space consisting of all points to the left of a plane parallel to the $xz$ - plane and 9 units to the right of it.

3. Write an inequality to describe the region consisting of all points between (but not on) the spheres of radius 5 and 7 centered at the origin.

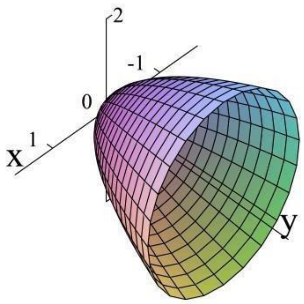
4 Write an equation or inequality that represents all points on or inside a circular cylinder of radius 9 with the $y$-axis as its axis

5 Identify the surface: $25x^2 + z^2 = 100 + 100y^2$

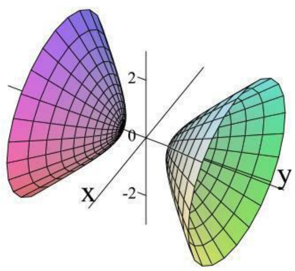
6 Classify the surface: $25x^2 + y^2 - z^2 - 6y + 6z = 0$

7 Find an equation for the surface obtained by rotating the parabola $x = z^2$ about the $x$-axis.

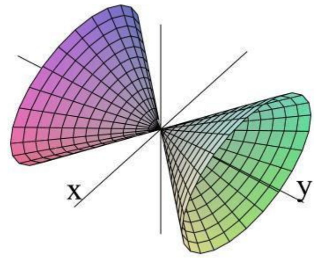
8 Match the equation with its graph: $y = x^2 + z^2$



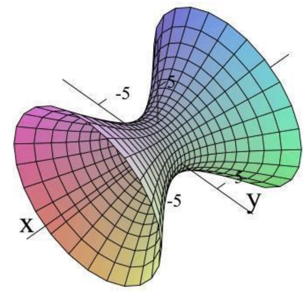

GRAPH 1




GRAPH II




GRAPH III




GRAPH IV

## 10.7: Space curves

1. Find $r'(t)$ for the function given by $r(t) = 2\mathbf{i} + \sin t\,\mathbf{j} + \cos t\,\mathbf{k}$

2. If $\mathbf{r}(t) = (t + t^2)\mathbf{i} + (2 + t^3)\mathbf{j} + t^4\mathbf{k}$, evaluate $\int_0^1 \mathbf{r}(t)dt$ .

3. Find a vector function, $\mathbf{r}(t)$, that represents the curve of intersection of the two surfaces

(a) The paraboloid $y = 2x^2 + z^2$ and the parabolic cylinder $x = z^2$
(b) The cylinder $x^2 + z^2 = 9$ and the plane $y = 3$.
(c) The cylinder $y^2 + z^2 = 4$ and the plane $x + 2y - 3z = 1$

4. Find parametric equations for the tangent line to the curve with parametric equations $x = 1 + 6\sqrt{t},\ y = t^3 - t,\ z = t^3 + t$ at the point (7, 0, 2).

## 10.8: Arc Length

1. Find the length of the curve given by $r(t) = 7\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k},\ \ 0 \le t \le 3$
2. Find the length of the curve $r(t) = \langle 2+3t, 1-4t, -4+3t \rangle$ from (5,-3,-3) to (20,-23, 14)
3. For the curve given by $r(t) = \left\langle \frac{t^3}{3}, \frac{t^2}{2}, t \right\rangle$, find the unit tangent vector.
4. Find the arc length of the curve given by $r(t) = <\cos 2t, \sin 2t, t>$ where t is going from 0 to $2\pi$.

**10.9: Motion in space**

1. A particle moves with position function $r(t) = 2\sqrt{2}\, t\mathbf{i} + e^{2t}\mathbf{j} + e^{-2t}\mathbf{k}$. Find the acceleration of the particle.
2. Find the velocity of a particle with the given position function $\mathbf{r}(t) = 13e^{15t}\mathbf{i} + 10e^{-18t}\mathbf{j}$
3. A particle moves with position function $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 2t\mathbf{k}$. Find the normal component of the acceleration vector.
4. A particle moves with position function $\mathbf{r}(t) = (9t - 3t^3 - 2)\mathbf{i} + 9t^2\mathbf{j}$. Find the tangential component of the acceleration vector.
5. A projectile is fired from ground level with an initial speed of 550 m/sec and an angle of elevation of 30 degrees. Use that the acceleration due to gravity is 9.8 $m/\sec^2$ .
   Find (a) The range of the projectile (b) The maximum hieght of the projectile
   (c) The speed with which the projectile hits the ground.

6. A body of mass 6 kg moves in a (counterclockwise) circular path of radius 10 meters, making one revolution every 8 seconds. You may assume the circle is in the xy-plane, and so you may ignore the third component.
   (a). Compute the centripetal force acting on the body. (b) Compute the magnitude of the force.

7 Given that the acceleration vector is
$\boldsymbol{a}(t) = (-25\cos(t))\boldsymbol{i} + (-25\sin(t))\boldsymbol{j} + (2t)\boldsymbol{k}$
The initial velocity is $\boldsymbol{v}(0) = \boldsymbol{i} + \boldsymbol{k}$ and the initial position vector is $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$
Find the
(a) velocity vector $\boldsymbol{v}(t)$
(b) the position vector $\boldsymbol{r}(t)$

8 A particle with mass 3 kg moves with a position function $\boldsymbol{r}(t) = 3\cos(2t)\boldsymbol{i} + t^2\boldsymbol{j}$
What force acts on the particle at time $t = 0$ ?

# Answers

**10.1 3-D Coordinate system**

1. (a) 3, (b)2, (c) 5, (d) $\sqrt{34}$ (e)$\sqrt{13}$, (f) $\sqrt{29}$

2. Center (1,0,-5), radius $\frac{\sqrt{106}}{2}$

3. (2,1,0) on the xy-plane (0,1,3) on the yz-plane, and (2,0,3) on the xz-plane.
4. $(x-1)^2 + (y-2)^2 + (z-3)^2 = 14$
5. C(2,4,-1) is 1 away from the xy-plane.
6. $(x-1)^2 + (y-4)^2 + (z-7)^2 \leq 16$

## 10.2: Vectors

1. 20.4
2. Tension = 61.98 N
3. (a) $\langle \frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}} \rangle$ (b) $\langle \frac{5}{\sqrt{30}}, \frac{10}{\sqrt{30}}, \frac{25}{\sqrt{30}} \rangle$ (c) $\langle -\frac{3}{\sqrt{30}}, -\frac{6}{\sqrt{30}}, -\frac{15}{\sqrt{30}} \rangle$
4. (a) $< 4,0,-2 >$ (b) $\langle 16,-3,-7 \rangle$ (c) $\sqrt{10}$
5. (6,7)
6. $s = -3, \quad t = 2$

## 10.3: Dot Product

1. $x = -2, \quad x = -4$
2. $\pi/4$
3. 12.52
4. $\langle \frac{56}{5}, \frac{28}{5} \rangle$
5. (a) 90 J (b) $\langle \frac{45}{47}, \frac{135}{94}, \frac{405}{94} \rangle$ (c) $\langle \frac{237}{47}, \frac{-511}{94}, \frac{65}{94} \rangle$
6. 4800

## 10.4 : Cross Product/Scalar Triple Product

1 $\frac{1}{\sqrt{3}}\boldsymbol{i} - \frac{1}{\sqrt{3}}\boldsymbol{j} - \frac{1}{\sqrt{3}}\boldsymbol{k}$

2. $| \mathbf{u} \times \mathbf{v} | = 35$
3. 18
4. $\frac{11\sqrt{2}}{2}$
5. Down

## 10.5 Equations of Lines & Planes

1. $\frac{\sqrt{30}}{6}$
2. $27x + 4y + 32z = -33$
3. $x = 2 - t,\ y = -2 + 2t,\ z = 4 + 5t$
4. $x = -2 + 4t,\ y = 1 + 2t,\ z = 1 + 4t$
5. $6x - 66y + 78z = -132$
6. $3x + 7y + 18z = 55$
7. $x = -6 + 7t,\ y = 9 + 3t,\ z = 3 - 7t$
8. $(-7,\ -6,\ 7)$
9. $-3x - 27y + 9z = -27$

10. (4,5,5)

11. 1.47 radians

12. $\frac{5}{\sqrt{14}}$

**10.6 : Cylinders & Quadric Surfaces**

1. a. $(x+1)^2+(y-2)^2+(z-1)^2=69$, b. $(x+1)^2+(y-2)^2=68$
2. $y<9$
3. $25 < x^2 + y^2 + z^2 < 49$
4. $x^2 + z^2 \leq 81$
5. The surface is a hyperboloid of one sheet with axis the $y$-axis.
6. A cone with axis parallel to the $z$-axis and vertex (0, 3, 3).
7. $y^2+z^2=x$
8. GRAPH I

## 10.7: Space curves

1. $r'(t)=\cos t\,\mathbf{j}-\sin t\,\mathbf{k}$
2. $\frac{5}{6}\mathbf{i}+\frac{9}{4}\mathbf{j}+\frac{1}{5}\mathbf{k}$
3. (a) $\mathbf{r}(t) = \langle t^2, 2t^4 + t^2, t\rangle$ (b) $\mathbf{r}(t) = \langle 3\cos t, 3, 3\sin t\rangle$
   (c) $\mathbf{r}(t) = \langle -4\cos t + 6\sin t + 1, 2\cos t, 2\sin t\rangle$
4. $x = 3t + 7,\ \ y = 2t,\ \ z = 4t + 2$

**10.8 : Arc Length**

1. $\frac{1}{27}(85^{3/2}-8)$
2. $5\sqrt{34}$
3. $\langle t^2, t, 1\rangle/\sqrt{t^4+t^2+1}$

4. $2\pi\sqrt{5}$

**10.9 : Motion in space**

1. $a(t) = 4e^{2t}\mathbf{j} + 4e^{-2t}\mathbf{k}$
2. $v = 195e^{15t}\,\mathbf{i} - 180e^{-18t}\,\mathbf{j}$
3. 2
4. $18t$
5. (a) 26731.91 m (b) 3858.42 m (c) 550 m/sec
6.
   (a) $\boldsymbol{F} = -\frac{15\pi^2}{4}\cos\left(\frac{\pi}{4}t\right)\boldsymbol{i} - \frac{15\pi^2}{4}\sin\left(\frac{\pi}{4}t\right)\mathbf{j}$
   (b) $15\pi^2/4$
7. (a) $(-25\sin t + 1)\boldsymbol{i} + (-25\cos t + 25)\boldsymbol{j} + (t^2 + 1)\boldsymbol{k}$
   (b) $(25\cos t + t - 24)\boldsymbol{i} + (-25\sin t + 25t + 1)\boldsymbol{j} + \left(\frac{1}{3}t^3 + t + 1\right)\boldsymbol{k}$
8. $\langle -117.6,\ 19.6\rangle$