MAT 578 Real Analysis II Course Announcement

Professor: John Quigg **Semester:** Fall 2025

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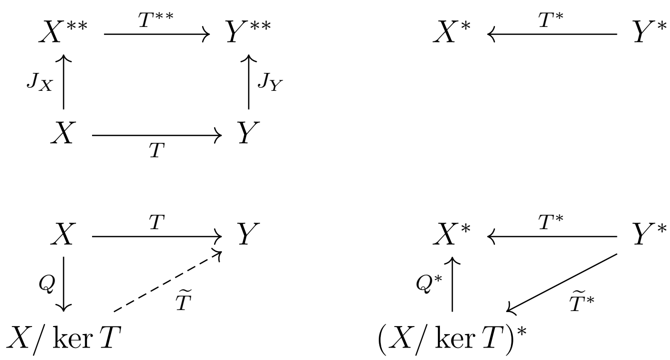

Classes: Tuesday and Thursday 10:30–11:45 in EDB L1-34

Description: Copied from the ASU web page for MAT 578:

> Locally convex, normed, and Hilbert spaces. Linear operators, spectral theory, and application to classical analysis.

But more informatively: This is the first half of a year-long sequence in functional analysis, which is the study of vector spaces equipped with a compatible topology, and continuous linear maps between them. Topics may include, but are not limited to: Banach spaces, Hilbert spaces, bounded linear maps, locally convex spaces, and duality. Some of the main theorems we will cover are: Open Mapping Theorem, Closed Graph Theorem, Uniform Boundedness Principle, Hahn–Banach Theorem, and Krein–Milman Theorem. To some extent, the syllabus will depend on the interests and abilities of the participants.

In the spring semester, the continuation will be a course on operator theory and spectral theory, using the C^*-algebra approach (the “little” Gelfand–Naimark Theorem).

Note: the sequence MAT 578–578 helps prepare one for the Comprehensive Exam in Functional Analysis (but there’s a bit extra required: the first 3 chapters of Murphy’s book).

Prerequisites: Copied from the ASU web page for MAT 578:

> Degree- or nondegree-seeking graduate student.

For admission to the Math PhD program, this means advanced calculus — at some institutions this course is called “introduction to real analysis" — (equivalent to MAT 371 at ASU) and linear algebra (equivalent to MAT 342). The specific advanced calculus requirement comprises a rigorous development of calculus on the real line, with complete proofs, and where students become proficient at constructing proofs of correct mathematical statements and counterexamples to incorrect ones, and how to tell the difference. The linear algebra requirement comprises abstract vector spaces over the real and complex numbers, linear transformations, and inner products.

But this is inadequate — it just means you've had linear algebra and advanced calculus in order to be admitted to the grad math program. Logically, the prerequisite for functional analysis would be the real analysis sequence MAT 570–571, or some knowledge of measure theory (and instructor approval). Much, but not all, of the course will be intelligible without a background in abstract measure theory. For someone willing to do some background reading on measure theory, the minimal prerequisite would be a course on metric space topology (such as MAT 472). And the introduction to Lebesgue integration given in MAT 473 would go a long way toward compensating for an absence of MAT 570–571.

However, I want to emphasize that, regardless of your background, if you are at all interested in this course, please contact me at quigg@asu.edu or WXLR 728.

Textbook: There is no required text — instead, I will post lecture notes. But suggested references include:

- J.B. Conway, "A Course in Functional Analysis", 2nd ed., Springer-Verlag, 1990.
- G.B. Folland, "Real Analysis", 2nd ed., Wiley, 1999.
- G.K. Pedersen, "Analysis now", Springer-Verlag, 1989.
- W. Rudin, "Functional Analysis", 2nd ed., McGraw-Hill, 1991.