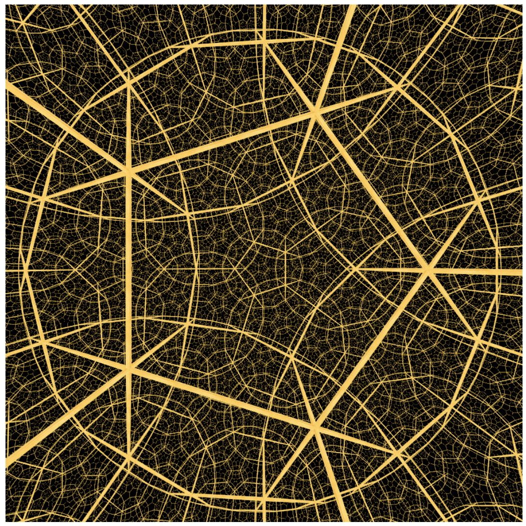
Arizona State University
MAT 598, Fall 2025
Instructor: Julien Paupert
TTh 1:30-2:45

Advanced Geometric Structures



This class will provide an introduction to locally homogeneous geometric structures on manifolds. Klein proposed in the celebrated Erlangen program that a *geometry* on a space X is provided by a transitive action of a Lie group G on X. Ehresmann and later Thurston studied manifolds M with geometric structures locally modelled on such a geometry (G, X), or (G, X)-structures. Examples include classical metric and uniformizable structures, where M is given as a global quotient of X under the covering action of a discrete group of isometries acting on X, as well as more flexible and exotic structures. We will study some general properties of such structures, families of examples and relations between them such as transitions between different geometries on a fixed manifold.

1 Topics

- Generalities on (G, X)-structures; holonomy and developing map
- Uniformizable metric structures
- Model spaces: constant curvature spaces, symmetric spaces
- Uniformization of surfaces
- Model geometries in dimension 3 and geometrization of 3-manifolds
- Convex projective structures
- Affine structures
- CR-spherical structures

2 Bibliography

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