MAT 267: Calculus III For Engineers

Test 1 Review

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1. If $P = (2, -1, 0)$, $Q = (4, 1, 1)$ and $R = (4, -5, 4)$, find all three side lengths of the triangle PQR.

2. Let $P = (2, 1, 4)$ and $Q = (4, 3, 10)$.
   a) Find the midpoint between P and Q.
   b) Find the equation of the sphere that containts P and Q as opposite points on its diameter.

3. Find the equation of the sphere centered at $(1, -2, 3)$, with a radius of 5.

4. Find the equation of the largest sphere possible that lies entirely in the first octant and is centered at $(3, 2, 5)$.

5. Find the equation of the sphere that passes through the origin and is centered at $(3, 2, 1)$.

6. Show that the following equation represents a sphere and find its center and radius:
   $$2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1.$$

7. If $P = (3, 1, 3)$ and $\overrightarrow{PQ} = \langle 1, -1, -2 \rangle$, then find the coordinates of Q.

8. Find the components of
   $$\mathbf{v} = 3(\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) - 6(\mathbf{i} + 2\mathbf{j}) - 10\mathbf{k} + \mathbf{j}.$$

9. Find the magnitude of $\mathbf{v} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

10. Find the unit vector that points from $P = (1, 2)$ to $Q = (4, 6)$.

11. If $\mathbf{v} = \langle 2, -4, 4 \rangle$, find the vector that is anti-parallel to $\mathbf{v}$ and has half its magnitude.

12. Find the unit vector in $\mathbb{R}^2$ that makes an angle of $\theta = 5\pi/6$ with the $+x$-axis.

13. A river that runs north and south has a current that flows at 5 mph southbound. A boat leaves the west shore, travelling at 20 mph. At what angle (with respect to east) should the boat be pointed so that it will travel directly east when crossing the river?

14. Find all values of x such that $\mathbf{v} = \langle 2x, -x, 16 \rangle$ is orthogonal to $\mathbf{w} = \langle 5, x, -1 \rangle$.

15. If $|\mathbf{u}| = 3$ and $\mathbf{u} \cdot \mathbf{v} = -1$, find the value of $\mathbf{u} \cdot (2\mathbf{u} + 5\mathbf{v})$.

16. Let $\mathbf{F} = \langle 3, 4, 1 \rangle$ and $\mathbf{D} = \langle -1, 2, 5 \rangle$.
    a) Find the angle between $\mathbf{F}$ and $\mathbf{D}$.
    b) Find vectors $\mathbf{p}$ and $\mathbf{n}$ such that $\mathbf{F} = \mathbf{p} + \mathbf{n}$, where $\mathbf{p}$ is parallel to $\mathbf{D}$ and $\mathbf{n}$ is orthogonal to $\mathbf{D}$.

17. A force vector $\mathbf{F} = \langle 2, 3, -9 \rangle$ N acts on a particle that moves from $P = (1, 2, 4)$ m to $Q = (5, 3, 1)$ m. How much work was done by the force?

18. Find the two unit vectors that are orthogonal to both $\mathbf{u} = \langle 0, 1, 2 \rangle$ and $\mathbf{v} = \langle 1, -2, 3 \rangle$.

19. Find the area of the triangle with vertices $(1, 1, 2)$, $(2, 3, 1)$ and $(-1, 0, 3)$.

20. Suppose $|\mathbf{u}| = 6$ and $|\mathbf{v}| = 2$, and the angle between them is $\theta = \pi/3$. Find the value of $|\mathbf{u} \times \mathbf{v}|$.

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21. Find the equation for the plane that contains the points $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 4)$.

22. Find the vector, parametric and symmetric equations of the line that passes through $(1, 1, 1)$ at $t = 0$ and $(3, 2, 2)$ at $t = 2$.

23. Find an equation for the line of intersection of the planes $x + y + z = 0$ and $x - y - z = 2$.

24. Find the point where the plane $2x - y + z = 2$ intersects the line $\mathbf{r}(t) = \langle 2 - t, 1 + 3t, 4t\rangle$.

25. Find an equation of the plane through the point $(5, 5, 0)$ and is parallel to the plane $x - y - 2z = 2$.

26. A particle travels along a straight line, and is at the point $(0, 1, -1)$ at $t = 1$ and at the pont $(7, 5, 3)$ at $t = 4$. Find the parametric equations for its trajectory.

27. Find the unit tangent vector of $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t\rangle$ at $t = 2$.

28. Find parametric equations for the tangent line of the curve $\mathbf{r}(t) = \langle t^3, t^2 - t, t^4 - 1\rangle$ and the point $(8, 2, 15)$.

29. Find parametric equations for the curve of intersection of the plane $x + y - z = 3$ and the parabolic cylinder $z = x^2$.

30. The helix $\langle \cos t, \sin t, t\rangle$ intersects the curve $\langle 1 + t, t^2, t^3\rangle$ at the point $(1, 0, 0)$. Find the angle (in radians) of intersection of these two curves.

31. Suppose $\mathbf{r}'(t) = \langle 6t, \cos(2t), e^{4t}\rangle$ and $\mathbf{r}(0) = \langle 1, 4, -1\rangle$. Find the vector equation for $\mathbf{r}(t)$.

32. Evaluate $\int_0^4 (t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k})\, dt$.

33. Find the length of the curve $\mathbf{r}(t) = \langle 4t + 1, 3t, 8t + 7\rangle$, $1 \le t \le 4$.

34. Find the length of the curve $\mathbf{r}(t) = \langle t, 3\cos t, 3\sin t\rangle$, $-5 \le t \le 5$.

35. Find the **TNB** vectors of the helix $\mathbf{r}(t) = \langle 2\cos(3t), 5t, 2\sin(3t)\rangle$ at $t = \pi$.

36. A particle has a position function $\mathbf{r}(t) = \langle t\ln t, t, e^{-t}\rangle$. Find its velocity, speed and acceleration when $t = 1$.

37. A spaceship is observed to have a position function $\mathbf{r}(t) = \langle 2t^2, 2t, t^2 - 6t\rangle$. At what time is its speed at a minimum?

38. A projectile is launched from the origin at a speed of 50 m/s and at an angle of $\theta = 60°$ above the $+x$-axis.
a) How long is it in the air?
b) What is the maximum height it reaches?
c) What is the x coordinate where it lands?

39. Find the position vector of a particle with acceleration $\mathbf{a}(t) = \langle 2t, \sin t, \cos 2t\rangle$, with an initial velocity $\mathbf{v}(0) = \langle 1, 0, 0\rangle$ and initial position $\mathbf{r}(0) = \langle 0, 1, 0\rangle$. If the particle has a mass of $m = 3$, find the magnitude of the force acting on it at $t = \pi/2$.

40. A planet is observed to have a circular orbital radius of $R = 3 \times 10^8$ km and it makes a full revolution around its star every 430 (Earth) days. What is the magnitude of its centripetal acceleration in units of m/s^2? State the answer in scientific notation with 3 significant figures.

Answers

1. $PQ = 3,\ PR = 6,\ QR = \sqrt{45}$
2. a) midpoint: $(3, 2, 7)$
   b) sphere: $(x-3)^2 + (y-2)^2 + (z-7)^2 = 11$
3. $(x-1)^2 + (y+2)^2 + (z-3)^2 = 25$
4. $(x-3)^2 + (y-2)^2 + (z-5)^2 = 4$
5. $(x-3)^2 + (y-2)^2 + (z-1)^2 = 14$
6. $(x-2)^2 + y^2 + (z+6)^2 = \frac{81}{2} \Rightarrow$ center $= (2, 0, -6)$, radius $= 9/\sqrt{2}$
7. $Q = (4, 0, 1)$
8. $\mathbf{v} = \langle -3, -17, 17 \rangle$
9. $|\mathbf{v}| = \sqrt{11}$
10. $\frac{1}{5}\langle 3, 4 \rangle$
11. $\langle -1, 2, -2 \rangle$
12. $\frac{1}{2}\langle -\sqrt{3}, 1 \rangle$
13. $\theta = \sin^{-1}(1/4) = 0.25268$ rad $= 14.48°$
14. $x = 2,\ x = 8$
15. 13
16. a) $\theta = 1.2046$ rad $= 69.02°$
    b) $\mathbf{p} = \langle -1/3, 2/3, 5/3 \rangle$, $\mathbf{n} = \langle 10/3, 10/3, -2/3 \rangle$
17. 38 J
18. $\pm\frac{1}{\sqrt{54}}\langle 7, 2, -1 \rangle$
19. $\sqrt{11}/2$
20. $6\sqrt{3}$
21. $4x + 2y + z = 4$
22. vector: $\langle 1+t, 1+t/2, 1+t/2 \rangle$
    parametric: $x = 1+t,\ y = 1+t/2,\ z = 1+t/2$
    symmetric: $x - 1 = 2(y-1) = 2(z-1)$
23. $\langle 1, -1+2t, -2t \rangle$
24. $(1, 4, 4)$
25. $x - y - 2z = 0$
26. $\frac{1}{3}\langle 7t-7, 4t-1, 4t-7 \rangle$
27. $\frac{1}{9}\langle 4, 8, 1 \rangle$

28. $x = 8 + 12t, \quad y = 2 + 3t, \quad z = 15 + 32t$

29. $x = t, \quad y = 3 + t^2 - t, \quad z = t^2$

30. $\pi/2$

31. $\mathbf{r}(t) = \langle 3t^2 + 1, -\frac{1}{2}\sin(2t) + 4, \frac{1}{4}(e^{4t} - 5)\rangle$

32. $\langle 8, \frac{64}{3}, 64\rangle$

33. $3\sqrt{89}$

34. $10\sqrt{10}$

35. $\mathbf{T} = \frac{1}{\sqrt{61}}\langle 0, 5, -6\rangle, \;\; \mathbf{N} = \langle 1, 0, 0\rangle, \;\; \mathbf{B} = \frac{1}{\sqrt{61}}\langle 0, -6, -5\rangle$

36. $\mathbf{v} = \langle 1, 1, -e^{-1}\rangle, \;\; |\mathbf{v}| = \sqrt{2 + e^{-2}}, \;\; \mathbf{a} = \langle 1, 0, e^{-1}\rangle$

37. $t = 0.6$

38. a) 8.83 s, b) $y_{max} = 95.57$ m, c) $x_{max} = 220.70$ m

39. $\mathbf{r}(t) = \langle t^3/3 + t, t - \sin t + 1, 1/4 - \cos(2t)/4\rangle, \quad |\mathbf{F}| = 3\sqrt{\pi^2 + 2}$

40. 8.58×10^{-3} m/s^2