# MAT 211 EXAM 1 REVIEW

## Section 4.1 Review: System of Linear Equations

1. Solve the following system using any means necessary (you may use your calculator). Express your answer as an ordered pair $(x, y)$. If there is no solution, enter NO SOLUTION. If the system is dependent, express your answer in terms of *x*, where *y* = *y*(*x*). Make sure to show all your work.

   a. $\begin{Bmatrix} 2x + 3y = 1 \\ 5x + 4y = 6 \end{Bmatrix}$ b. $\begin{Bmatrix} 4x - 8y = 20 \\ 2x - 4y = 10 \end{Bmatrix}$ c. $\begin{Bmatrix} -2x + 2y = 4 \\ 3x - 3y = -5 \end{Bmatrix}$

2. You manage an ice cream factory that makes two flavors: Creamy Vanilla and Continental Mocha. Into each quart of Creamy Vanilla go 2 eggs and 3 cups of cream. Into each quart of Continental Mocha go 1 egg and 3 cups of cream. You have in stock 550 eggs and 1050 cups of cream. How many quarts of each flavor should you make in order to use up all the eggs and cream?

## Section 6.1/6.2 Linear Programming

3. The area of a parking lot is 600 square meters. A car requires 6 square meters. A bus requires 30 square meters. The attendant can handle only 60 vehicles. If a car is charged $2.50 and a bus $7.50. How many of each should be accepted to maximize Profit?

   a. **Set up** for the solution of this Linear Programming (**LP**) problem.
   b. Using part (a), how many of each should be accepted to maximize Profit? What is the maximum profit?

4. Every week, Pecos Oil Company must process at least 30,000 barrels of low-grade oil and 20,000 barrels of high-grade oil to supply its gasoline stations. The company's refinery in Mentone can process 6000 barrels of low-grade oil and 2500 barrels of high-grade oil per day, at a cost of $40,000 per day. The company's other refinery in Reeves can process 3000 barrels of low-grade oil and 5000 barrels of high-grade oil per day, at a cost of $25,000 per day. How many days per week should the company operate each refinery in order to minimize the cost? What is that minimum cost?

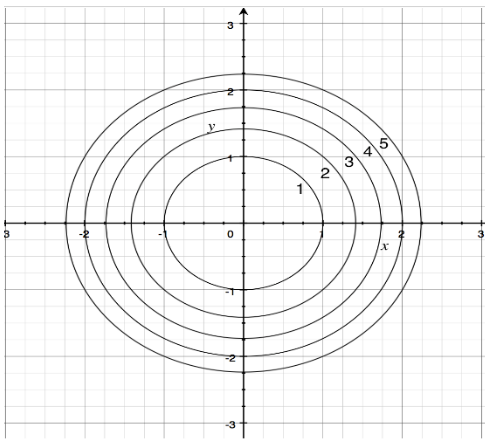
   a. **Set up** for the solution of this Linear Programming (**LP**) problem.
   b. Using part (a), how many days per week should the company operate each refinery in order to minimize the cost? What is that minimum cost?

## Section 15.1/15.2 Functions of Several Variables/ Partial Derivatives

5. Let $f(x, y, z) = 1.5 + 2.3x - 1.4y - 2.5z$. Complete the following sentences.
   a. *f* _______________ by________________units for every 1 unit increase in *x*.
   b. *f* _______________ by________________units for every 1 unit increase in *y*.
   c. _______________ by 2.5 units for every ____________________________.

6. Brand Z's annual sales are affected by the sales of related products X and Y as follows: Each $1 million increase in sales of brand X causes a $2.5 million decline in sales of brand Z, whereas each $1 million increase in sales of brand Y results in an increase of $23 million in sales of brand Z. Currently, brands X and Y are each selling for $2 million per year and brand Z is selling $62 million per year. Model the sales of brand Z using a **linear function**. (Let $z$ = annual sales of $Z$ (in millions of dollars), $x$ = annual sales of $X$ (in millions of dollars), and $y$ = annual sales of $Y$ (in millions of dollars).)

7. Refer to the following plot of some **level curves** of $f(x, y) = c$ for $c$ = 1, 2, 3, 4 and 5.




Estimate

a) $f(1,-1) \approx$ _________ .

b) $f(-1,2) \approx$ _________.

c) $f(-1.5,-0.5) \approx$ ______ .

8. Given $f(x, y) = x^2y + 3y + x^2y^3 - 4x$, evaluate the following and simplify your result completely.
   a) $f(a, 4)$ b) $f_{xx}(x, y)$. c) $f_{xy}(x, y)$ d) $f_{yy}(x, y)$. $e)$ $f_{yx}(x, y)$.

9. Given $f(x, y, z) = xe^{2y+3z}$ .
   Find $a.$ $f_x(1, -1, 1)$ $b.$ $f_y(1, -1, 1)$ c. $f_z(1, -1, 1)$

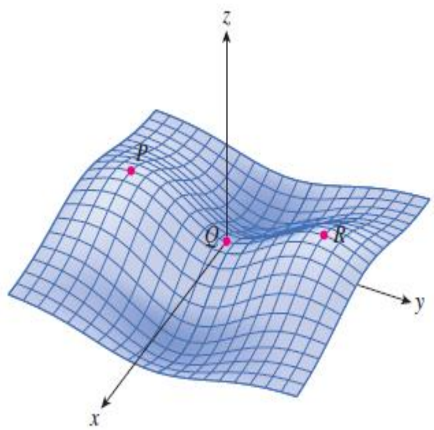
10. Your weekly cost (in dollars) to manufacture x *cars* and y *trucks* is

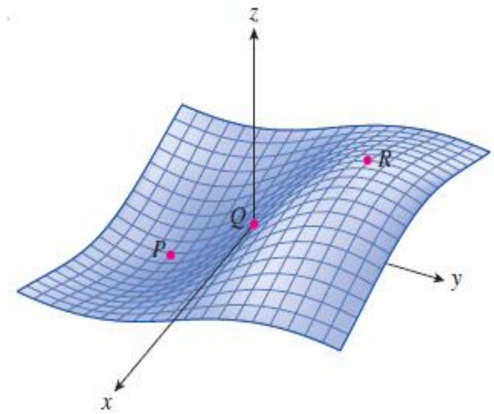
$$C(x, y) = 240{,}000 + 6000x + 4000y - 20xy.$$

   a. Compute the marginal cost of manufacturing *cars* at a production level of 10 cars and 20 trucks.
   b. Compute the marginal cost of manufacturing *trucks* at a production level of 10 cars and 20 trucks.

**Section 15.3 Maximum and Minimum Values**

11. Consider the following provided graphs.

a.



b.



Classify each labeled point on each graph as one of the following:
*Relative maximum, Relative minimum, Saddle point or neither a relative extrema or saddle point.*

12. Given a function $f(x, y) = xy^2 - 6x^2 - 3y^2$, determine the following

a) Find the critical point, $(x, y)$ of the function, $f$.
b) Classify the critical point found in part (a) as Relative Minimum, Relative Maximum or Saddle point. (Hint: Use the Second Derivative Test.)

13. Given a function $f(x, y) = xy - 5y + x^2 + y^2 - 10x$, determine the following

a) Find the critical point, $(x, y)$ of the function, $f$.
b) Classify the critical point found in part (a) as Relative Minimum, Relative Maximum or Saddle point. (Hint: Use the Second Derivative Test.)

**Section 15.4 Constrained Optimization**

14. Use *substitution* method to solve the following optimization problem.
Find the maximum value of $f(x, y, z) = 1 - x^2 - y^2 - z^2$ subject to $z = 5y$.
Also find the corresponding point $(x, y, z)$.

15. Use *Lagrange multipliers* to solve the given optimization problem.
Find the maximum value of $z = f(x, y) = xy$ subject to $3x + y = 60$.
Express your answer as an ordered pair $(x, y, z)$

16. Use *Lagrange multipliers* to solve the given optimization problem.
Find the minimum value of $z = f(x, y) = x^2 + y^2$ subject to $x + 2y = 10$.
Express your answer as an ordered pair $(x, y, z)$.

17. A consumer's utility function is given by $f(x, y) = (3x + 1)y$ where $x$ is the quantity of good X that is bought and $y$ is the quantity of good Y that is bought. The price of good X is \$3 while the price of good Y is \$4. If the consumer has \$31 to spend on good X and Y, use *Lagrange multipliers* to find the consumer's optimal utility level (i.e. calculate the consumer's optimization problem of dividing her money between goods X and Y in the way that maximizes their utility).

18. You want to fence in a rectangular vegetable patch. The fencing for the east and west sides costs \$4 per foot, and the fencing for the north and south sides costs only \$2 per foot. I have a budget of \$80 for the project. Use *Lagrange multipliers* to find the dimensions of the vegetable patch with the largest area I can enclose? What is the largest area?

## EVT Extreme Value Theorem

19. Find the absolute maximum and absolute minimum of the function:

$$z = f(x, y) = 4x + 6y - x^2 - y^2$$

subject to the constraints: $\begin{cases} 0 \le x \le 4 \\ 0 \le y \le 5 \end{cases}$

[Hint: *Sketch* the region with the constraints above and follow the steps below to complete the problem]

a) Find all the corner points of the region.
*b)* A critical point(s) that lie within the region.
c) All the boundary points when subjecting $f(x, y)$ to the boundary constraints.
d) Find all corresponding z-values in the blanks provided and state the absolute maximum and minimum values of $f$ and their corresponding (x, y) points.

ABSOLUTE MIN: ________________ ABSOLUTE MAX: ________________

20. Repeat question 19 with the following function and constraints inequalities

$$z = f(x, y) = (x - 10)^2 + (y - 20)^2 + 150$$

subject to the constraints: $2x + y \le 20,\ x \ge 0,\ y \ge 0.$

21. Repeat question 19 with the following function and constraints inequalities

$$z = f(x, y) = 4x + 10y + 20$$

subject to the constraints: $y \le 36 - x^2,\ y \ge 0.$

# Answers

1. a. Independent system with one unique solution, (2, -1).
   b. Dependent system with an infinite number of solutions, ($x$, $\frac{x-5}{2}$).
   c. No Solution

2. 200 quarts of Creamy Vanilla and 150 quarts of Continental Mocha

3. a. Let $c$ be the # of cars and $b$ be the # of buses. Let P be the profit in dollars.

   Maximize P= 2.50$c$ +7.50$b$
   Subject to $c + b \leq 60$
   $6c + 30b \leq 600$
   $c \geq 0, b \geq 0$

   b. 50 cars and 10 buses; Profit= $200

4. a. Let $m$ be the # of days per week for operating the Mentone refinery
   Let $r$ be the # of days per week for operating the Reeves refinery.
   Let C be the cost in dollars.

   Minimize C= 40,000$m$ + 25,000$r$
   Subject to $6{,}000m + 3{,}000r \geq 30{,}000$
   $2{,}500m + 5{,}000r \geq 20{,}000$
   $m \geq 0, r \geq 0$

   c. Mentone for 4 days, Reeves for 2 days; minimum cost of $210,000

5. a. $f$ __increases__ by __2.3__ units for every 1 unit increase in $x$.
   b. $f$ __decreases__ by __1.4__ units for every 1 unit increase in $y$.
   c. __$f$ decreases__ by 2.5 units for every __1 unit increase in z__ .

6. z = $-2.5x + 23y - 21$

7. *a.* $f(1,-1) \approx$ __2__ . b. $f(-1,2) \approx$ __5__ . c. $f(-1.5,-0.5) \approx$ __2.5__ .

8. a. $f(a,4)$=$68a^2 - 4a + 12$ b. $f_{xx}(x,y)$= $2y + 2y^3$ c. $f_{xy}(x,y)$= $2x + 6xy^2$
d. $f_{yy}(x,y)$=$6x^2y$ e. $f_{yx}(x,y)$= $2x + 6xy^2$ [Note: $f_{xy}(x,y)$= $f_{yx}(x,y)$]

9. a. $f_x(1,-1,1) = e$ b. $f_y(1,-1,1) = 2e$ c. $f_z(1,-1,1) = 3e$

10. a. $5600 per car b. $3800 per truck

11. a. P: Relative maximum, Q: Saddle point, R: Relative maximum
b. P: Relative minimum, Q: Neither, R: Relative maximum.

12. a. Critical points are (0,0), (3,6), (3, -6)
b. (0,0): Relative maximum, (3,6) and (3, -6): Saddle point.

13. a. Critical point is (5,0)
b. (5,0): Relative minimum

14. Maximum value of *f*=1 and occurs at (0,0,0)

15. (10,30,300)

16. (2,4,20)

17. 5 units of good X, 4 units of good Y.

18. (5,10,50)

19. (a) (0,0), (4,0), (0,5), (4,5)
(b) (2,3)
(c) (0,3), (4,3), (2,0), (2,5)
(d) Absolute maximum value of *f*=13 and occurs at (2,3). Absolute minimum value of *f*=0 and occurs at (0,0) and (4,0).

20. (a) (0,0), (10,0), (0,20)
(b) No critical point(s) in the interior of the region
(c) All 3 corner points and (2,16).
(d) Absolute maximum value of *f*=650 and occurs at (0,0). Absolute minimum value of *f*=230 and occurs at (2,16).

21. (a) (-6,0), (6,0)
(b) No critical point(s) in the interior of the region
(c) All 2 corner points and (1.2,34.56).
(d) Absolute maximum value of *f*=370.4 and occurs at (1.2, 34.56) and absolute minimum value of *f*= -4 and occurs at (-6,0).