**Concepts**:

- Understand the structure of an argument and be able to distinguish between valid and invalid arguments.
- Understand the basic rules of inference and recognize them in arguments.
- Recognize invalid arguments by name and by justification.
- Apply the rules of inference to build formal proofs for valid arguments.

**Problems**:

1. Fill in the blanks in the statements below:

   (a) An argument is valid if and only if . . . .
   (b) The argument with premises $p$ and $p \to q$ and conclusion $q$ is called . . .
   (c) A fallacy is defined as . . .
   (d) If from premises $q$ and $p \to q$ someone concludes $p$, that the reasoning is called the fallacy of . . . .

2. Check whether the following statements are true or false. Briefly justify your answer.

   (a) An argument with all premises being false is valid.
   (b) The rule of inference with hypotheses $p \vee q$ and $r \vee \neg q$, and conclusion $p \vee r$ is called Addition.
   (c) The order of premises in an argument matters.
   (d) In a formal proof, we can use a premise more than once.

3. Identify the following statements as a valid argument or a fallacy. If the argument is valid, identify the argument form.

   (a) I can clean my house by myself or hire cleaners. If I clean myself, I save money. If I hire cleaners, I can read a book. Therefore, I save money, or I can read a book.
   (b) If the Flash restrained his enemies first instead of taunting them, he would always have the advantage of surprise. Having the advantage of surprise means winning every fight. Therefore, if the Flash restrained his enemies first instead of taunting them, he would win every fight.
   (c) If the sun is shining, I'm happy. I'm unhappy. Therefore, the sun is not shining.
   (d) If the sun is shining, I'm happy. The sun is not shining. Therefore, I'm unhappy.

4. Identify the following as a valid argument or a fallacy. If the argument is valid, identify the argument form.

   *Erin spends the summer in Europe but she spends the school year (fall, winter and spring) in the United States to go to school. Therefore, Erin spends the year in Europe or in the United States.*

   (a) Modus Ponens
   (b) Moduls Tollens
   (c) Resolution
   (d) Hypothetical syllogism
   (e) None of these

5. Identify the following statements as a valid argument or a fallacy. If the argument is valid, identify the argument form.

   (a) If the weather is nice, then I go to the beach. If I go to the beach, then I feel happy. Therefore, if the weather is nice, then I feel happy.

   (b) 2 is an even number. 2 is prime. Therefore 2 is an even prime number.

   (c) Today I go for a walk and call my sister. Therefore, I call my sister today.

   (d) If you can vote, then you are 18 or older. Kylie cannot vote. Therefore, Kylie is younger than 18.

   (e) If you can vote, then you are 18 or older. Kylie is younger than 18. Therefore, Kylie cannot vote.

6. Identify the following statements as a valid argument or a fallacy. If there was more than one rule of inference used, select the last one used.

   (a) Every computer science student takes a discrete mathematics course. Sydney is a computer science student. Therefore Sydney takes a discrete mathematics course.

   (b) Every bug is an insect. Elephants are not bugs. Trombone is an elephant. Therefore, Trombone is not an insect.

   (c) Every bug is an insect. Elephants are not insects. Trombone is an elephant. Therefore, Trombone is not a bug.

   (d) Every bug is an insect. Wizzy-Fizz is an insect. Therefore, Wizzy-Fizz is a bug.

7. Use the rules of inference to prove $p \wedge s$, given the following premises. Write your solution as a numbered sequence of statements. Identify each statement as either a premise, a conclusion that follows according to a rule of inference from previous statements, or being logically equivalent to a previous statement. State the rule of inference or the law of logical equivalence and refer by number to the previous statement(s) that the rule of inference or law of logical equivalence relied on.

| | | |
|---|---|---|
| 1. | $\neg r$ | premise |
| 2. | $s$ | premise |
| 3. | $q \vee r$ | premise |
| 4. | $q \rightarrow p$ | premise |

8. Give a formal proof to show that the argument below is valid.

$$\begin{array}{c} a \wedge b \\ a \rightarrow \neg (b \wedge c) \\ d \rightarrow c \\ \hline \therefore \quad \neg d \end{array}$$

9. Consider the following argument:

   *All wizards can do magic. Muggles can't do magic. Peter is a muggle.*
   *Therefore, Peter is not a wizard.*

   Several students at Hogwarts formalized this argument (without using magic). Here is Ron's attempt:

   Let's define the predicates:
   $W(x)$="$x$ is a wizard", $M(x)$="$x$ is a muggle", $A(x)$="$x$ can do magic"

   Universe of discourse: all people (magical and non-magical)

1. $\forall x(W(x) \to A(x))$ (assumption)
2. $\forall x(M(x) \to \neg A(x))$ (assumption)
3. $M(\text{Peter})$ (assumption)
4. $M(\text{Peter}) \to \neg A(\text{Peter})$ (Universal instantiation of 2)
5. $\neg A(\text{Peter})$ (Modus Ponens from 3 and 4)
6. $W(\text{Peter}) \to A(\text{Peter})$ (universal instantiation of 1)
7. $\therefore \neg W(\text{Peter})$ (Modus Tollens from 5 and 6)

Is Ron's argument valid or invalid?

10. Consider the following argument:

*All wizards can do magic. Muggles can't do magic. Peter is a muggle.*
*Therefore, Peter is not a wizard.*

Several students at Hogwarts formalized this argument (without using magic). Here is Neville's attempt:

Let's define the predicates: $W =$ "is a wizard", $M =$ "is a muggle", $A =$ "can do magic."

Universe of discourse: all people (magical and non-magical)

1. $W(x) \to A(x)$ (assumption)
2. $M(x) \to \neg A(x)$ (assumption)
3. $M(\text{Peter})$ (assumption)
4. $M(\text{Peter}) \to \neg A(\text{Peter})$ (Universal Instantiation of 2)
5. $\neg A(\text{Peter})$ (Modus Ponens from 3 and 4)
6. $W(\text{Peter}) \to A(\text{Peter})$ (Universal Instantiation of 1)
7. $\therefore \neg W(\text{Peter})$ (Modus Tollens from 5 and 6)

Is Neville's argument valid or invalid?

11. Formalize the argument by using the given predicates and then rewriting the argument as a numbered sequence of statements. Identify each statement as either a premise, a conclusion that follows according to a rule of inference from previous statements, or being logically equivalent to a previous statement. State the rule of inference or the law of logical equivalence and refer by number to the previous statement(s) on which the rule of inference or the law of logical equivalence relied.

*Bunnies eat vegetables. Vinny is a bunny. Carrots are vegetables. Therefore Vinny eats carrots.*

Use the following predicates: $E(x, y)$="$x$ eats $y$", $B(x)$="$x$ is a bunny", $V(y)$= "$y$ is a carrot",

$x$ represents animals and $y$ represents vegetables.

**Solutions:**

1. Fill in the blanks in the statements below:

   (a) An argument is valid if and only if $p_1 \wedge p_2 \wedge \ldots \wedge p_n \to c$ is a tautology, where $p_1, p_2, \ldots, p_n$ are the premises of the argument and $c$ is its conclusion. That is, the truth of premises implies the truth of the conclusion.

   (b) The argument with premises $p$ and $p \to q$ and conclusion $q$ is called Modus Ponens.

   (c) A fallacy is defined as an invalid argument.

   (d) If from premises $q$ and $p \to q$ someone concludes $p$, that reasoning is called the fallacy of affirming the conclusion.

2. Check whether the following statements are true or false. Briefly justify your answer.

   (a) An argument with all premises false is valid.

   True. See Problem 1a.

   (b) The rule of inference with hypotheses $p \vee q$ and $r \vee \neg q$, and conclusion $p \vee r$ is called Addition.

   False. It is the rule called Resolution.

   (c) The order of premises in an argument matters.

   False. The conjunction of the premises is commutative, thus their order does not matter.

   (d) In a formal proof, we can use a premise more than once.

   True. There is no rule how many times a premise can be used in an argument.

3. Identify the following as a valid argument or a fallacy. If the argument is valid, identify the argument form.

   (a) I can clean my house by myself or hire cleaners. If I clean myself, I save money. If I hire cleaners, I can read a book. Therefore, I save money, or I can read a book.

   Resolution.

   (b) If the Flash restrained his enemies first instead of taunting them, he would always have the advantage of surprise. Having the advantage of surprise means winning every fight. Therefore, if the Flash restrained his enemies first instead of taunting them, he would win every fight.

   Hypothetical Syllogism.

   (c) If the sun is shining, I'm happy. I'm unhappy. Therefore, the sun is not shining.

   Modus Tollens.

   (d) If the sun is shining, I'm happy. The sun is not shining. Therefore, I'm unhappy.

   Fallacy of Denying the Hypothesis.

4. Identify the following as a valid argument or a fallacy. If the argument is valid, identify the argument form.

   *Erin spends the summer in Europe but she spends the school year (fall, winter and spring) in the United States to go to school. Therefore, Erin spends the year in Europe or in the United States.*

   (a) Modus Ponens
   (b) Moduls Tollens
   (c) Resolution
   (d) Hypothetical Syllogism
   (e) None of these

   If we rewrite the two premises as conditionals, then the resolution rule has the following form, which is more intuitive in some cases:

$$\begin{array}{l} \neg p \to q \\ \underline{\;\; p \to r \;\;} \\ \therefore \;\; q \vee r \end{array}$$

   Let $p =$"it is fall, winter or spring", $q =$"Erin is in Europe", $r =$"Erin is in the United States." Then

$$\begin{array}{ll} \neg p \to q & \text{“Erin spends the summer in Europe.”} \\ \underline{p \to r} & \underline{\text{“Erin spends the school year (fall, winter and spring) in the United States.”}} \\ \therefore \;\; q \vee r & \text{“Therefore, Erin spends the year in Europe or in the United States.”} \end{array}$$

5. Identify the following as a valid argument or a fallacy. If the argument is valid, identify the argument form.

   (a) If the weather is nice, then I go to the beach. If I go to the beach, then I feel happy. Therefore, if the weather is nice, then I feel happy.

   Hypothetical Syllogism.

   (b) 2 is an even number. 2 is prime. Therefore, 2 is an even prime number.

   Conjunction.

   (c) Today I go for a walk and call my sister. Therefore, I call my sister today.

   Simplification.

   (d) If you can vote, then you are 18 or older. Kylie cannot vote. Therefore, Kylie is younger than 18.

   Fallacy of Denying the Hypothesis.

   (e) If you can vote, then you are 18 or older. Kylie is younger than 18. Therefore, Kylie cannot vote.

   (Universal) Modus Tollens.

6. Identify the following as a valid argument or a fallacy. If there was more than one rule of inference used, select the last one used.

(a) Every computer science student takes a discrete mathematics course. Sydney is a computer science student. Therefore Sydney takes a discrete mathematics course.

Valid. (Universal) Modus Ponens.

(b) Every bug is an insect. Elephants are not bugs. Trombone is an elephant. Therefore, Trombone is not an insect.

Fallacy of Denying the Hypothesis.

(c) Every bug is an insect. Elephants are not insects. Trombone is an elephant. Therefore, Trombone is not a bug.

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(d) Every bug is an insect. Wizzy-Fizz is an insect. Therefore, Wizzy-Fizz is a bug.

Fallacy of Affirming the Conclusion.

7. Use the rules of inference to prove $p \land s$, given the following premises. Write your solution as a numbered sequence of statements. Identify each statement as either a premise, a conclusion that follows according to a rule of inference from previous statements, or being logically equivalent to a previous statement. State the rule of inference or the law of logical equivalence and refer by number to the previous statement(s) that the rule of inference or law of logical equivalence relied on.

| | | |
|---|---|---|
| 1. | $\neg r$ | premise |
| 2. | $s$ | premise |
| 3. | $q \lor r$ | premise |
| 4. | $q \to p$ | premise |
| 5. | $q$ | from (1) and (3) by Disjunctive Syllogism |
| 6. | $p$ | from (4) and (5) by Modus Ponens |
| 7. | $p \land s$ | from (2) and (6) by Conjunction |

8. Give a formal proof to show that the argument below is valid.

$$\begin{array}{c} a \land b \\ a \to \neg(b \land c) \\ d \to c \\ \hline \therefore \quad \neg d \end{array}$$

| | | |
|---|---|---|
| 1. | $a \land b$ | premise |
| 2. | $a \to \neg(b \land c)$ | premise |
| 3. | $d \to c$ | premise |
| 4. | $a$ | from 1 by Simplification |
| 5. | $\neg(b \land c)$ | from 2 and 4 by Modus Ponens |
| 6. | $\neg b \lor \neg c$ | from 5 by DeMorgan's Law |
| 7. | $b$ | from 1 by Simplification |
| 8. | $\neg(\neg b)$ | from 7 by Double Negation |
| 9. | $\neg c$ | from 6 and 8 by Disjunctive Syllogism |
| 10. | $\neg d$ | from 3 and 9 by Modus Tollens |

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Let's define the predicates:
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Universe of discourse: all people (magical and non-magical)

1. $\forall x(W(x) \to A(x))$ (assumption)
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4. $M(\text{Peter}) \to \neg A(\text{Peter})$ (Universal Instantiation of 2)
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6. $W(\text{Peter}) \to A(\text{Peter})$ (Universal Instantiation of 1)
7. $\therefore \neg W(\text{Peter})$ (Modus Tollens from 5 and 6)

Is Ron's argument valid or invalid?

Ron's argument is valid

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6. $W(\text{Peter}) \to A(\text{Peter})$ (Universal Instantiation of 1)
7. $\therefore \neg W(\text{Peter})$ (Modus Tollens from 5 and 6)

Is Neville's argument valid or invalid?

Neville's argument is invalid. Premises 1 and 2 should be universally quantified statements. They are predicates, not propositions.

11. Formalize the argument by using the given predicates and then rewriting the argument as a numbered sequence of statements. Identify each statement as either a premise, a conclusion that follows according to a rule of inference from previous statements, or being logically equivalent to a previous statement. State the rule of inference or the law of logical equivalence and refer by number to the previous statement(s) on which the rule of inference or law of logical equivalence relied.

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| | | |
|---|---|---|
| 1. | $\forall x \forall y((B(x) \land V(y)) \to E(x, y))$ | premise |
| 2. | $B(Vinny)$ | premise |
| 3. | $V(Carrot))$ | premise |
| 4. | $(B(Vinny) \land V(Carrot)) \to E(Vinny, Carrot)$ | Universal Instantiation of 1 |
| 5. | $B(Vinny) \land V(Carrot)$ | Conjunction from 2 and 3 |
| 6. | $\therefore E(Vinny, Carrot)$ | Modus Ponens from 4 and 5 |