Concepts:

- Translate English sentences involving "for all" or "there exists" into formal mathematical statements by defining appropriate predicates and selecting appropriate quantifiers and logical connectives.
- Express domain restricted universally quantified statements as the unrestricted universal quantification of a conditional.
- Express domain restricted existentially quantified statements as the unrestricted existential quantification of a conjunction.
- Identify variables as bound or free.
- Identify the scope of a quantifier.
- Negate quantified statements by applying the De Morgan's Laws for quantified statements.
- Recognize statements as logically equivalent or nonequivalent that involve universal and existential quantification and conjunction and disjunction.
- Determine the truth value of mathematical statements involving nested quantifiers.

Problems:

1. Check whether the following statements are true or false. The domain is the set of real numbers.

   (a) "For all real numbers x, if $x = 1$, then $2x + 3 = 2$" is a proposition.

   (b) $\forall x(x = 2 \wedge x = 3 \rightarrow x = 0)$.

   (c) $\exists x(x > 1 \rightarrow x^2 < 0)$.

   (d) $\forall x(x > 1 \rightarrow x^2 < 0)$.

   (e) $\forall x(x > 1 \rightarrow x^2 > 1)$.

2. Check whether the following statements are true or false where P and Q are predicates in one variable.

   (a) $\exists x P(x) \wedge \exists x Q(x) \rightarrow \exists x(P(x) \wedge Q(x))$.

   (b) $\exists x(P(x) \wedge Q(x)) \rightarrow \exists x P(x) \wedge \exists x Q(x)$.

   (c) $\forall x P(x) \wedge \forall x Q(x) \rightarrow \forall x(P(x) \wedge Q(x))$.

   (d) $\forall x(P(x) \vee Q(x)) \rightarrow \forall x P(x) \vee \forall x Q(x)$.

   (e) $\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$.

   (f) $\exists x(P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists x Q(x)$.

   (g) $\forall x(P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$.

(h) $\forall x(P(x) \vee Q(x)) \equiv \forall x P(x) \vee \forall x Q(x)$.

3. Select which of the following configurations of truth values for P and Q on the domain $\{0, 1\}$ provides a counterexample to $\forall x(P(x) \vee Q(x)) \to \forall x P(x) \vee \forall x Q(x)$. Note that $\{0, 1\}$ is the set with two numbers, 0 and 1 as two elements.

(a)

x	$P(x)$	$Q(x)$
0	T	F
1	T	F

(b)

x	$P(x)$	$Q(x)$
0	T	F
1	F	T

(c)

x	$P(x)$	$Q(x)$
0	F	T
1	T	F

(d)

x	$P(x)$	$Q(x)$
0	F	T
1	F	T

(e)

x	$P(x)$	$Q(x)$
0	F	F
1	F	F

4. Which symbolic expression is the correct representation of the following statements? Explain your reasoning.

(a) *"Every ASU student who takes discrete mathematics will learn valid arguments."*

Universe of discourse: all ASU students.
$D(x)$: "x takes discrete mathematics course."
$A(x)$: "x learns valid arguments."

- $\forall x(D(x) \wedge A(x))$
- $\forall x D(x) \to A(x)$
- $\forall x(D(x) \to A(x))$

(b) *"There is an ASU student who takes discrete mathematics and knows valid arguments."*

Universe of discourse: all ASU students.
$D(x)$: "x takes discrete mathematics course."
$A(x)$: "x knows valid arguments."

- $\exists x D(x) \wedge A(x)$
- $\exists x(D(x) \wedge A(x))$

- $\exists x(D(x) \rightarrow A(x))$

(c) *"Pat loves exactly one person."*

Universe of discourse: all people.
$L(x, y)$: "x loves y"

- $\exists y L(Pat, y) \wedge \forall z(L(Pat, z) \rightarrow z = y)$
- $\exists y(L(Pat, y) \wedge \forall z(L(Pat, z) \rightarrow z = y))$
- $\exists y \forall z(L(Pat, y) \wedge (L(Pat, z) \rightarrow z = y))$

5. Assume x is a real number.

(a) The domain restricted existential statement $\exists x > 0(x^2 > 2)$ is equivalent to ...

i. $\exists x(x > 0 \wedge x^2 > 2)$
ii. $\exists x(x > 0 \rightarrow x^2 > 2)$

(b) The domain restricted universal statement $\forall x > 0(x^2 > 2)$ is equivalent to ...

i. $\forall x(x > 0 \wedge x^2 > 2)$
ii. $\forall x(x > 0 \rightarrow x^2 > 2)$

6. Let H be the predicate defined on the set of all people by $H(x)$ = "x is happy." Which of the following statements are acceptable translations of the mathematical statement $\exists x H(x)$ into standard English?

(a) Happy people exist.
(b) There exists a person x such that x is happy.
(c) There exists a happy person.
(d) There are happy people.
(e) There is at least one happy person.

7. Let B and H be predicates defined on the set of all animals by B = "is a bunny" and H = "hops." Use quantifiers and the given predicates to express the following statement.
"Every bunny hops but not every animal that hops is a bunny."

8. Select all sets I for which the quantified statement $\forall x \exists y(x + y < 1)$ is true if I is both the domain for x and y.

(a) $I = \{0\}$ is the set with just the one number, 0 as an element.
(b) $I = \{0, 1\}$ is the set with two numbers, 0 and 1 as two elements.
(c) $I = (0, 1)$ is the continuum of real numbers strictly between 0 and 1.
(d) $I = [0, 1]$ is the continuum of real numbers from 0 to 1, including 0 and 1.

9. Show that $\exists y \forall x((2x-1)(y-3)=0)$ is a true statement, where the domain of discourse is the set of positive integers.

10. What is the negation of $\exists x > 1(x^2 > 2)$ where x is a real number?

    (a) $\forall x > 1(x^2 \leq 2)$
    (b) $\forall x \leq 1(x^2 \leq 2)$
    (c) $\exists x > 1(x^2 \leq 2)$
    (d) $\exists x \leq 1(x^2 \leq 2)$

11. Express the negation of the following statement in its simplest form. The final answer should not contain the negation symbol.

    $\forall x \exists y(y > 0 \rightarrow (-2 \leq x < 6))$

12. Which of the following is the negation of the statement $\forall y \exists x((x > y) \rightarrow (x + y > 0))$.

    (a) $\exists y \forall x((\neg x > \neg y) \vee (x + y \leq 0))$
    (b) $\exists y \forall x((x > y) \wedge (\neg x + \neg y > 0))$
    (c) $\exists y \forall x((x > y) \wedge (x + y \leq 0))$
    (d) $\exists y \forall x((x \leq y) \wedge (x + y \leq 0))$
    (e) none of these

13. Which of the following is the negation of "All MAT 243 students know logic"?

    (a) No other students besides MAT 243 students know logic.
    (b) No MAT 243 students know logic.
    (c) All MAT 243 students do not know logic.
    (d) There is a student who has not taken MAT 243 and who knows logic.
    (e) All students who have not taken MAT 243 know logic.
    (f) There is a student who has not taken MAT 243 and does not know any subject other than logic.
    (g) There is at least one MAT 243 student who doesn't know logic.

14. Select all statements that are always true for any predicate P on any (nonempty) domain of two variables. Show your reasoning.

    (a) $\forall x \exists y P(x,y) \equiv \exists y \forall x P(x,y)$

    (b) $\forall x \exists y P(x,y) \rightarrow \exists y \forall x P(x,y)$

    (c) $\exists x \forall y P(x,y) \rightarrow \forall y \exists x P(x,y)$

    (d) $\forall x \forall y P(x,y) \equiv \forall y \forall x P(x,y)$

    (e) $\exists x \exists y P(x,y) \equiv \exists y \exists x P(x,y)$

(f) $\forall x \forall y P(x, y) \rightarrow \exists x \exists y P(x, y)$

(g) $\exists y \exists x P(x, y) \rightarrow \forall y \forall x P(x, y)$

(h) $\forall x \forall y P(x, y) \rightarrow \forall x \exists y P(x, y)$

(i) $\forall x \forall y P(x, y) \rightarrow \exists y \forall x P(x, y)$

15. Let the domain of discourse be the positive integers, chose the correct mathematical representation of the statement "there is a smallest positive integer."

(a) $\exists x \forall y (x < y)$
(b) $\exists x \forall y (x \leq y)$
(c) $\forall y \exists x (x < y)$
(d) $\forall y \exists x (x \leq y)$

16. (a) Check whether the following statements are true or false. The domain for all variables is the set of integers.

i. $\forall x \exists y (x + 2y = 1)$
ii. $\exists y \forall x (x + 2y = 1)$
iii. $\exists x \forall y (x + 2y = 1)$
iv. $\forall y \exists x (x + 2y = 1)$

(b) Check whether the following statements are true or false. The domain for x is the set $\{0\}$ and the domain for y is the set $\{-1, 1\}$.

i. $\forall x \exists y (x^2 + y^2 = 1)$
ii. $\exists x \forall y (x^2 + y^2 = 1)$
iii. $\forall x \forall y (x^2 + y^2 = 1)$
iv. $\exists x \exists y (x^2 + y^2 = 1)$

Solutions:

1. Check whether the following statements are true or false. The domain is the set of real numbers.

   (a) "For all real numbers x, if $x = 1$, then $2x + 3 = 2$" is a proposition.

   True. "if $x = 1$, then $2x + 3 = 2$" is a predicate in variable x and the given statement is the universal quantification of this predicate, and hence it is a proposition. It is a false proposition, since for $x = 1$, "if $x = 1$, then $2x + 3 = 2$" is false.

   (b) $\forall x(x = 2 \wedge x = 3 \rightarrow x = 0)$.

   True. Note that "$x = 2 \wedge x = 3$" is false for any value of x. Thus, for any x, the conditional statement "$x = 2 \wedge x = 3 \rightarrow x = 0$" is true, and hence the universal statement is true.

   (c) $\exists x(x > 1 \rightarrow x^2 < 0)$.

   True. We pick $x = 0$. For $x = 0$, "$x > 1 \rightarrow x^2 < 0$" is true, and hence the existential statement is true.

   (d) $\forall x(x > 1 \rightarrow x^2 < 0)$.

   False. For $x = 2$, the premise is true, but the conclusion is false. Thus, "$x > 1 \rightarrow x^2 < 0$" is false for $x = 2$, and hence the universal statement is false.

   (e) $\forall x(x > 1 \rightarrow x^2 > 1)$.

   True. Assume x is an arbitrary real number.
   Case 1: If $x \leq 1$, the premise is false, thus the conditional statement is true.
   Case 2: If $x > 1$, multiply both sides of the inequality $x > 1$ by x to obtain $x^2 > x > 1$, which implies $x^2 > 1$. Thus, the conditional statement is true for $x > 1$.
   We have shown that for any real number x, "$(x > 1 \rightarrow x^2 > 1)$" is true.

2. Check whether the following statements are true or false where P and Q are predicates in one variable.

   (a) $\exists xP(x) \wedge \exists xQ(x) \rightarrow \exists x(P(x) \wedge Q(x))$.

   False. Consider the counterexample of $P(x) =$ "x is odd" and $Q(x) =$ "x is even", with the universe of discourse the set of integers. Then the statements $\exists xP(x)$ and $\exists xQ(x)$ are both true, hence the premise $\exists xP(x) \wedge \exists xQ(x)$ is true. However, the conclusion $\exists x(P(x) \wedge Q(x))$ is false, since there is no integer both even and odd.

   (b) $\exists x(P(x) \wedge Q(x)) \rightarrow \exists xP(x) \wedge \exists xQ(x)$.

   True. We will need to justify two possible cases.
   Case 1: If the premise is false, the conditional statement is true by default.
   Case 2: Assume the premise $\exists x(P(x) \wedge Q(x))$ is true, that is for $x = x_0$, $P(x) \wedge Q(x)$ is true. Then $P(x_0)$ is true and $Q(x_0)$ is true, and hence both statements $\exists xP(x)$ and $\exists xQ(x)$ are true. Thus, the conclusion $\exists xP(x) \wedge \exists xQ(x)$ is also true.

   (c) $\forall xP(x) \wedge \forall xQ(x) \rightarrow \forall x(P(x) \wedge Q(x))$.

   True. We will need to justify two possible cases.
   Case 1: If there exists an $x = x_0$ for which $P(x_0)$ or $Q(x_0)$ is false, then the premise $\forall xP(x) \wedge \forall xQ(x)$

is false, and consequently the conditional statement is true.
Case 2. If $\forall x P(x)$ is true and $\forall x Q(x)$ is true, then for each $x = x_0$, both $P(x_0)$ and $Q(x_0)$ are true, and hence $\forall x(P(x) \wedge Q(x))$ is true.

(d) $\forall x(P(x) \vee Q(x)) \to \forall x P(x) \vee \forall x Q(x)$.

False. Use the even/odd counterexample in part a.

(e) $\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$.

True. The existential quantifier distributes over the OR logical operator. We omit a rigorous proof of this statement. If you are interested in giving one, you have to prove the following two statements:
$\exists x(P(x) \vee Q(x)) \to \exists x P(x) \vee \exists x Q(x)$,
$\exists x P(x) \vee \exists x Q(x) \to \exists x(P(x) \vee Q(x))$.

(f) $\exists x(P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists x Q(x)$.

False. See part a.

(g) $\forall x(P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$.

True. The universal quantifier distributes over the AND logical operator. We omit a rigorous proof of this statement. If you are interested in giving one, you have to prove the following two statements:
$\forall x(P(x) \wedge Q(x)) \to \forall x P(x) \wedge \forall x Q(x)$,
$\forall x P(x) \wedge \forall x Q(x) \to \forall x(P(x) \wedge Q(x))$.

(h) $\forall x(P(x) \vee Q(x)) \equiv \forall x P(x) \vee \forall x Q(x)$.

False. See part d.

3. Select which of the following configurations of truth values for P and Q on the domain $\{0, 1\}$ provides a counterexample to $\forall x(P(x) \vee Q(x)) \to \forall x P(x) \vee \forall x Q(x)$. Note that $\{0, 1\}$ is the set with two numbers, 0 and 1 as two elements..

(a)

x	$P(x)$	$Q(x)$
0	T	F
1	T	F

In this case $\forall x(P(x) \vee Q(x))$ is true and $\forall x P(x) \vee \forall x Q(x)$ is true, so the above conditional statement is true. This scenario is not a counter example.

(b)

x	$P(x)$	$Q(x)$
0	T	F
1	F	T

Since $\forall x(P(x) \vee Q(x))$ is true but $\forall x P(x) \vee \forall x Q(x)$ is false, this is a counterexample for the above statement.

(c)

x	$P(x)$	$Q(x)$
0	F	T
1	T	F

Since $\forall x(P(x) \vee Q(x))$ is true but $\forall x P(x) \vee \forall x Q(x)$ is false, this is also a counterexample.

(d)

x	$P(x)$	$Q(x)$
0	F	T
1	F	T

In this case $\forall x(P(x) \vee Q(x))$ is true and $\forall x P(x) \vee \forall x Q(x)$ is true, so the above conditional statement is true. This scenario is not a counter example.

(e)

x	$P(x)$	$Q(x)$
0	F	F
1	F	F

In this case $\forall x(P(x) \vee Q(x))$ is false and $\forall x P(x) \vee \forall x Q(x)$ is false, so the above conditional statement is true. This scenario is not a counter example.

4. Which symbolic expression is the correct representation of the following statements? Explain your reasoning.

   (a) *"Every ASU student who takes discrete mathematics will learn valid arguments."*

   Universe of discourse: all ASU students.
   $D(x)$: "x takes discrete mathematics course."
   $A(x)$: "x learns valid arguments."

   - $\forall x(D(x) \wedge A(x))$

     Incorrect. This symbolic expression represents "All ASU students take discrete mathematics course and learn valid arguments."

   - $\forall x D(x) \rightarrow A(x)$

     Incorrect.This expression is not a statement. It is a predicate in x, since the x in $A(x)$ is a free variable.

   - $\forall x(D(x) \rightarrow A(x))$

     Correct.

   (b) *"There is an ASU student who takes discrete mathematics and knows valid arguments."*

   Universe of discourse: all ASU students.
   $D(x)$: "x takes discrete mathematics course."
   $A(x)$: "x knows valid arguments."

   - $\exists x D(x) \wedge A(x)$

     Incorrect. This expression is not a statement. It is a predicate in x, since the x in $A(x)$ is a free variable.

   - $\exists x(D(x) \wedge A(x))$

     Correct.

- $\exists x(D(x) \to A(x))$

  Incorrect. This symbolic expression represents "If there is an ASU student who takes discrete mathematics, then this student knows valid arguments."
  Take an ASU student for x who has not taken discrete mathematics (premise is false), then this conditional statement is true, even if nobody knows valid arguments in the discrete mathematics course.

(c) *"Pat loves exactly one person."*

Universe of discourse: all people.
$L(x, y)$: "x loves y"

- $\exists y L(Pat, y) \land \forall z(L(Pat, z) \to z = y)$

  Incorrect. This expression is not a statement, since the y is a free variable in $\forall z(L(Pat, z) \to z = y)$.

- $\exists y(L(Pat, y) \land \forall z(L(Pat, z) \to z = y))$

  Correct.

- $\exists y \forall z(L(Pat, y) \land (L(Pat, z) \to z = y))$

  Correct.

5. Assume x is a real number.

(a) The domain restricted existential statement $\exists x > 0(x^2 > 2)$ is equivalent to ...

i. $\exists x(x > 0 \land x^2 > 2)$ Correct.
ii. $\exists x(x > 0 \to x^2 > 2)$

(b) The domain restricted universal statement $\forall x > 0(x^2 > 2)$ is equivalent to ...

i. $\forall x(x > 0 \land x^2 > 2)$
ii. $\forall x(x > 0 \to x^2 > 2)$ Correct.

6. Let H be the predicate defined on the set of all people by $H(x)$ = "x is happy." Which of the following statements are acceptable translations of the mathematical statement $\exists x H(x)$ into standard English?

(a) Happy people exist.

Correct.

(b) There exists a person x such that x is happy.

Incorrect. We do not use variable names in standard English.

(c) There exists a happy person.

Correct.

(d) There are happy people.

Correct

(e) There is at least one happy person.

Correct.

7. Let B and H be predicates defined on the set of all animals by $B =$ "is a bunny " and $H =$ "hops." Use quantifiers and the given predicates to express the following statement.
"Every bunny hops but not every animal that hops is a bunny."

$\forall x\,(B(x) \rightarrow H(x)) \wedge \exists x\,(H(x) \wedge \neg B(x))$

8. Select all sets I for which the quantified statement $\forall x \exists y(x + y < 1)$ is true if I is both the domain for x and y.

(a) $I = \{0\}$ is the set with just the one number, 0, as an element.

True.

(b) $I = \{0, 1\}$ is the set with two numbers, 0 and 1, as two elements.

False. For $x = 1$, there is no y in I, for which $x + y < 1$.

(c) $I = (0, 1)$ is the continuum of real numbers strictly between 0 and 1.

True.

(d) $I = [0, 1]$ is the continuum of real numbers from 0 to 1, including 0 and 1.

False. For $x = 1$, there is no y in I, for which $x + y < 1$.

9. Show that $\exists y \forall x((2x - 1)(y - 3) = 0)$ is a true statement, where the domain of discourse is the set of positive integers.

Pick $y = 3$. Then $(2x - 1)(y - 3) = 0$ for all x.

10. What is the negation of $\exists x > 1(x^2 > 2)$ where x is a real number?

(a) $\forall x > 1(x^2 \leq 2)$ Correct.

$\neg(\exists x > 1(x^2 > 2)) \equiv \neg(\exists x(x > 1 \wedge (x^2 > 2))) \equiv \forall x \neg(x > 1 \wedge (x^2 > 2)) \equiv \forall x(x \leq 1 \vee (x^2 \leq 2)) \equiv \forall x(x > 1 \rightarrow (x^2 \leq 2)) \equiv \forall x > 1(x^2 \leq 2)$.

(b) $\forall x \leq 1(x^2 \leq 2)$

(c) $\exists x > 1(x^2 \leq 2)$

(d) $\exists x \leq 1(x^2 \leq 2)$

11. Express the negation of the following statement in its simplest form. The final answer should not contain the negation symbol.

$$\forall x \exists y (y > 0 \to (-2 \leq x < 6))$$

$\exists x \forall y \, (y > 0 \wedge (x < -2 \vee 6 \leq x))$.

Note that the negation of $-2 \leq x < 6$ is NOT $-2 > x \geq 6$. The later inequality would imply that $-2 > 6$, which is false.

The inequality $-2 \leq x < 6$ is equivalent to $-2 \leq x \wedge x < 6$, so $\neg(-2 \leq x \wedge x < 6) \equiv x < -2 \vee 6 \leq x$.

12. Which of the following is the negation of the statement $\forall y \exists x((x > y) \to (x + y > 0))$.

   (a) $\exists y \forall x((\neg x > \neg y) \vee (x + y \leq 0))$
   (b) $\exists y \forall x((x > y) \wedge (\neg x + \neg y > 0))$
   (c) $\exists y \forall x((x > y) \wedge (x + y \leq 0))$ Correct.
   (d) $\exists y \forall x((x \leq y) \wedge (x + y \leq 0))$
   (e) none of these

13. Which of the following is the negation of "All MAT 243 students know logic"?

   (a) No other students besides MAT 243 students know logic.
   (b) No MAT 243 students know logic.
   (c) All MAT 243 students do not know logic.
   (d) There is a student who has not taken MAT 243 and who knows logic.
   (e) All students who have not taken MAT 243 know logic.
   (f) There is a student who has not taken MAT 243 and does not know any other subject than logic.
   (g) There is at least one MAT 243 student who does not know logic. Correct.

   Let $P(x)$: "x knows logic" with domain of discourse all MAT 243 students at ASU. Then, symbolically, "All MAT 243 students know logic" is $\forall x P(x)$. The negation of this statement is $\exists x \neg P(x)$ with English translation "there is at least one MAT 243 student who does not know logic."

14. Select all statements that are always true for any predicate P on any (nonempty) domain of two variables. Show your reasoning.

   (a) $\forall x \exists y P(x, y) \equiv \exists y \forall x P(x, y)$

   False. Counterexample: $\forall x \exists y \, (2x + 5y = 7)$ is true, but $\exists y \forall x \, (2x + 5y = 7)$ is false on the set of integers, so they are not logically equivalent.

   (b) $\forall x \exists y P(x, y) \to \exists y \forall x P(x, y)$

   False. Use the same counterexample used in part a.

   (c) $\exists x \forall y P(x, y) \to \forall y \exists x P(x, y)$

   True. If the premise $\exists x \forall y P(x, y)$ is false, the conditional is true by default. Assume $\exists x \forall y P(x, y)$ is true. That is, there exists $x = x_0$ for which $P(x_0, y)$ for all y. Then $\forall y \exists x P(x, y)$ is also true, since for each y we can pick $x = x_0$ to satisfy the statement.

(d) $\forall x \forall y P(x, y) \equiv \forall y \forall x P(x, y)$

True. The proof is left for the reader.

(e) $\exists x \exists y P(x, y) \equiv \exists y \exists x P(x, y)$

True. The proof is left for the reader.

(f) $\forall x \forall y P(x, y) \rightarrow \exists x \exists y P(x, y)$

True. The proof is left for the reader.

(g) $\exists y \exists x P(x, y) \rightarrow \forall y \forall x P(x, y)$

False. The proof is left for the reader.

(h) $\forall x \forall y P(x, y) \rightarrow \forall x \exists y P(x, y)$

True. The proof is left for the reader.

(i) $\forall x \forall y P(x, y) \rightarrow \exists y \forall x P(x, y)$

True. The proof is left for the reader.

15. Let the domain of discourse be the positive integers, choose the correct mathematical representation of the statement "there is a smallest positive integer."

(a) $\exists x \forall y (x < y)$
(b) $\exists x \forall y (x \leq y)$ Correct.
(c) $\forall y \exists x (x < y)$
(d) $\forall y \exists x (x \leq y)$

16. (a) Check whether the following statements are true or false. The domain for all variables is the set of integers.

i. $\forall x \exists y (x + 2y = 1)$

False. For $x = 0$, there is no integer y for which $x + 2y = 1$.

ii. $\exists y \forall x (x + 2y = 1)$

False. There is no integer y for which $x + 2y = 1$ for all x.

iii. $\exists x \forall y (x + 2y = 1)$

False. There is no integer x for which $x + 2y = 1$ for all y.

iv. $\forall y \exists x (x + 2y = 1)$

True. Assume y is an arbitrary integer. Let $x = 1 - 2y$. Then x is an integer and $x + 2y = (1 - 2y) + 2y = 1$.

(b) Check whether the following statements are true or false. The domain for x is the set $\{0\}$, the domain for y is the set $\{-1, 1\}$.

i. $\forall x \exists y (x^2 + y^2 = 1)$

True. We only have one choice for x, which is $x = 0$. Then we could pick either $y = 1$ or $y = -1$ to make the statement true.

ii. $\exists x \forall y (x^2 + y^2 = 1)$

True. With $x = 0$, this statement will be true for all y in $\{-1, 1\}$.

iii. $\forall x \forall y (x^2 + y^2 = 1)$

True. The statement is true for $x = 0, y = 1$ and for $x = 0, y = -1$, which covers all the possibilities.

iv. $\exists x \exists y (x^2 + y^2 = 1)$

True.