APM 598: Stochastic Differential Equations and applications



- Instructor: Sebastien Motsch (email: smotsch@asu.edu)
- Class: Tu,Th 3:00-4:15pm (WXLRA 302)
- Office hours: Tu,Th 1:30-2:30pm (GWC 642)

Textbook: L. Evans, "An Introduction to Stochastic Differential Equations"

Supplementary: B. Oksendal, "Stochastic Differential Equations: An Intro. with Applications"

Requirements:

Ordinary Differential equations (MAT 475 or APM 501). It is also recommended to have some experience in programming (Python or Julia preferably) and in probability theory (e.g. STP 421 or APM 504).

Course Description

This course aims at introducing stochastic differential equations (SDEs):

$$\mathrm{d}X_t = b(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t.$$

The course will emphasize using computational techniques to discretize and simulate solutions to SDEs. This will help students become more comfortable with the notations and formalism of SDE.

The course will be divided in two parts. In the first part of the course, we will define Brownian motion and learn how to use it to add noise to *ordinary* differential equations. We will then explore the link between SDEs and partial differential equations using the Itô's formula. As an application, we will study the Ornstein-Uhlenbeck process (see figure above), which is also known as the Black-Scholes model in mathematical finance.

In the second part of the course, we will focus on pratical applications of SDEs starting with stochastic gradient descent (SGD). SGD is a popular algorithm for training machine learning models, which often requires solving high-dimensional minimization problems. Finally, we will also explore diffusion models, a recent application of SDEs to generative modeling. Diffusion models generate new samples, such as images, by "reversing" the dynamics of the Ornstein-Uhlenbeck process.

Grading

Homework	6	50%
Project	1	50%