School of Mathematical and Statistical Sciences
Arizona State University

M E M O R A N D U M

DATE: 3/29/2024

TO: Faculty and Students

FROM: Professor(s) Julien Paupert Brett Kotschwar
Chair/Co-Chairs of Jerry Magana
Defense for the MA in Mathematics
Committee Members Sharon Crook

DEFENSE ANNOUNCEMENT

Candidate: Jerry Magana

Defense Date: Wednesday, April 10, 2024

Defense Time: 4:00 PM

Virtual Meeting Link: https://asu.zoom.us/j/83916657931 In Person: WXLR 546 (Wexler Hall - Tempe)

Title: On the Classification of Hyperbolic Triangle Groups & Non-Arithmetic Lattices of Hyperbolic Orbifolds

Please share this information with colleagues and other students, especially those studying in similar fields. Faculty and students are encouraged to attend. The defending candidate will give a 40-minute talk, after which the committee members will ask questions. There may be time for questions from those in attendance. But, guests are primarily invited to attend as observers and will be excused when the committee begins its deliberations or if the committee wishes to question the candidate privately.

ABSTRACT
-See next page-

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ABSTRACT

The study of hyperbolic manifolds, and more generally hyperbolic orbifolds, is intimately bound to the study of discrete subgroups of the isometry group of hyperbolic n-space. In the wake of certain rigidity theorems due to Mostow et al., a new program of study has developed in recent decades for the characterization of hyperbolic manifolds by investigating certain invariants arising from the theory of numbers. Critical to the arithmetic study of hyperbolic manifolds are those discrete subgroups of the isometry group which have finite co-volume under the Haar metric, sometimes called lattices. These correlate to a particular tiling of hyperbolic space with a certain fundamental domain. The simplest non-trivial example of these for hyperbolic orbifolds are triangle groups.

These triangle groups, or more properly arithmetic Fuchsian triangle groups, were first classified by Takeuchi in 1983. In the proceeding manuscript, a concise introduction to the geometry of hyperbolic manifolds and orbifolds is put forth. The two primary invariants used in the study of the hyperbolic lattices, the invariant trace field and the invariant quaternion algebra, are then defined. Thereafter, a hyperbolic triangle group is constructed from the tessellation of the hyperbolic plane by hyperbolic triangles. A version of the classification theorem of arithmetic Fuchsian triangle groups is stated and proved. The paper concludes with a brief discussion regarding non-arithmetic lattices.