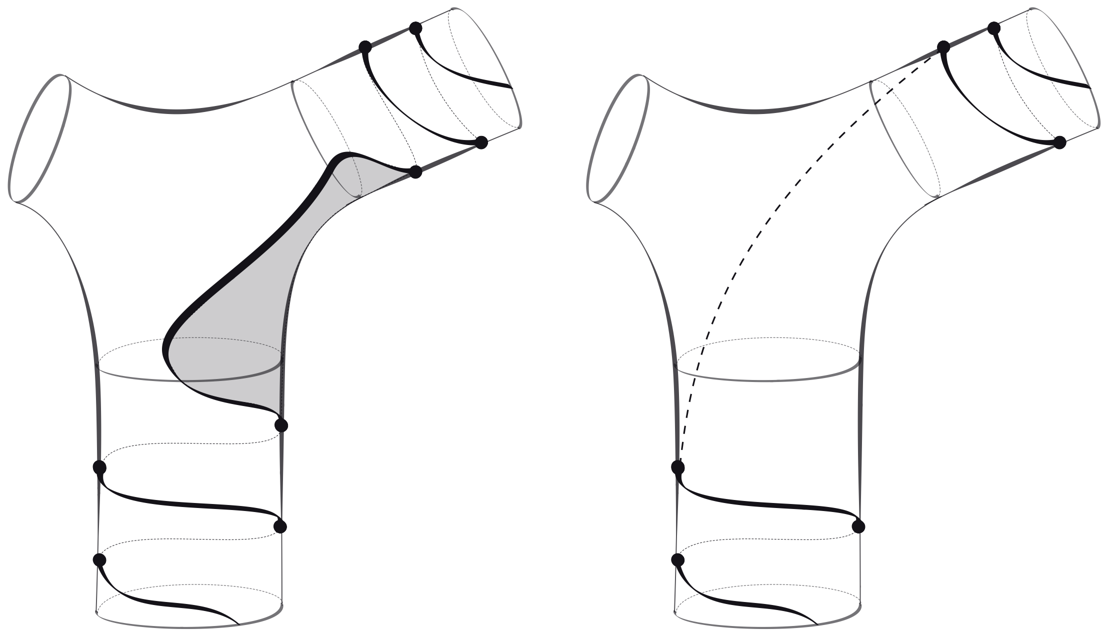
Arizona State University
Instructor: Hanh Vo

Topics: Curves on surfaces



Since the nineteenth century, the theory of surfaces has been extensively studied from the topological as well as the differential geometric point of view. Riemann first studied complex structures on closed compact surfaces (Riemann surfaces) and Poincaré-Koebe [1, 2] classified simply connected Riemann surfaces in 1907.

Closed curves play important roles in the study of surfaces. They were initially studied in the context of complex analysis. Dehn [3, 4] studied these objects from a topological and combinatorial perspective. He expressed closed curves as finite words in the generators of the fundamental group and introduced the word problem (that is the characterization of the identity element of the fundamental group), the conjugacy problem (that is how to determine conjugate elements), etc.

For a long time, mathematicians were interested in simple closed curves. Starting from the work of Dehn, many important results were discovered, for example, by Fenchel-Nielsen [5], and then especially by Thurston [6] from the 1980s. There have been many remarkable results about simple closed curves, for example, the work of McShane [7, 8] and Mirzakhani [9, 10, 11]. Recently, there has been an interest in understanding to what extent results about simple curves can be reproduced for non-simple curves for example, in the work of Erlandsson-Souto [12, 13].

This course is an introduction to surfaces and curves on surfaces. Topics include:

- Finite type surfaces and hyperbolic structures
- Closed curves and geodesics on hyperbolic surfaces

- Intersections of closed curves and the bigon criterion
- Mapping class groups
- Dehn twists

Prerequisites: The prerequisites are: Advanced Calculus (MAT 371 or equivalent) and Linear Algebra (MAT 342 or equivalent). Some familiarity with groups and topology (connectedness, compactness, quotient spaces) is preferable but not required. References will be given to learn the minimal notions of these topics as needed.

Course work: There will be no homework or exams in this class, rather students will write a final project on the topic of their choice, related to any of the topics discussed in class.

References for the class:

- A.F. Beardon; The Geometry of Discrete Groups, Graduate Texts in Mathematics Vol. 91, Springer, 1983.
- Peter Buser, Geometry and spectra of compact Riemann surfaces, Progr. Math., 106, Birkhäuser Boston, Boston, MA, 1992.
- Benson Farb and Dan Margalit. A primer on mapping class groups, volume 49 of Princeton Mathematical Series. Princeton University Press, Princeton, NJ, 2012.

Further reading:

[1] H. Poincaré. Sur l'uniformisation des fonctions analytiques. *Acta Math.*, 31(1):1–63, 1908.

[2] P. Koebe. Ueber die uniformisierung beliebiger analytischer kurven. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse*, 1907:191–210, 1907.

[3] M. Dehn. Über die Topologie des dreidimensionalen Raumes. *Math. Ann.*, 69(1):137–168, 1910.

[4] M. Dehn. Über unendliche diskontinuierliche Gruppen. *Math. Ann.*, 71(1):116–144, 1911.

[5] Werner Fenchel and Jakob Nielsen. *Discontinuous groups of isometries in the hyperbolic plane*, volume 29 of *De Gruyter Studies in Mathematics*. Walter de Gruyter & Co., Berlin, 2003. Edited and with a preface by Asmus L. Schmidt, Biography of the authors by Bent Fuglede.

[6] Albert Fathi, François Laudenbach, and Valentin Poénaru. *Thurston's work on surfaces*, volume 48 of *Mathematical Notes*. Princeton University Press, Princeton, NJ, 2012. Translated from the 1979 French original by Djun M. Kim and Dan Margalit.

[7] Greg McShane. *A remarkable identity for lengths of curves*. ProQuest LLC, Ann Arbor, MI, 1991. Thesis (Ph.D.)–University of Warwick (United Kingdom).

[8] Greg McShane. Simple geodesics and a series constant over Teichmuller space. *Invent. Math.*, 132(3):607–632, 1998.

[9] Maryam Mirzakhani. Simple geodesics and Weil-Petersson volumes of moduli spaces of bordered Riemann surfaces. *Invent. Math.*, 167(1):179–222, 2007.

[10] Maryam Mirzakhani. Growth of the number of simple closed geodesics on hyperbolic surfaces. *Ann. of Math. (2)*, 168(1):97–125, 2008.

[11] Maryam Mirzakhani. *Simple geodesics on hyperbolic surfaces and the volume of the moduli space of curves*. ProQuest LLC, Ann Arbor, MI, 2004. Thesis (Ph.D.)–Harvard University.

[12] Viveka Erlandsson and Juan Souto. Counting curves in hyperbolic surfaces. *Geom. Funct. Anal.*, 26(3):729–777, 2016.

[13] Viveka Erlandsson and Juan Souto. Mirzakhani's Curve Counting. 7 pages, November 2020.