MAT 210 Final Exam Review Questions Combined

Limits (sections 10.1, 10.3)

1. Calculate the limits of the following functions.
   a. $\lim_{x\to 2} 2x^3 - 2x + 5\sqrt{x+2}$
   b. $\lim_{x\to 3} \frac{x^2+x-6}{13x-26}$
   c. $\lim_{x\to 5} 3e^{x-5}$
   d. $\lim_{x\to -2} \frac{ax^2+2x+2}{3x-2}$ (a is a non-zero constant)
   e. $\lim_{x\to -1} \frac{ax^3-3x^2+3}{4x^3+1}$ (a is a non-zero constant)
   f. $\lim_{x\to 3} \frac{\mathrm{a}x-3}{\mathrm{b}x^2-1}$ (a and b are non-zero constants)

2. Calculate the limits of the following rational functions.
   a. $\lim_{x\to\infty} \frac{3\,x^3-3}{-13\mathrm{x}^3-4x^2-2}$
   b. $\lim_{x\to\infty} \frac{7\,x^2-5}{19\mathrm{x}^3-3x\,-7}$
   c. $\lim_{x\to\infty} \frac{-9\,x^5+5}{12\mathrm{x}^3-3x\,-7}$
   d. $\lim_{x\to\infty} \frac{-3}{11x^3}$
   e. $\lim_{x\to\infty} \frac{-3x^3}{11}$
   f. $\lim_{x\to-\infty} \frac{ax^3-3x+1}{bx^3-x^2}$ (a and b are non-zero constants)
   g. $\lim_{x\to-\infty} \frac{ax^4+x-100}{bx^3-3x+3}$ (a and b are positive non-zero constants)
   h. $\lim_{x\to\infty} \frac{ax^3+9}{bx^6-10x}$ (a and b are non-zero constants)
   i. $\lim_{x\to\infty} \frac{5ax^3-4cx+1}{-8x^2+10bx^3}$ (a, b, and c are non-zero constants)
3. Calculate the limit: $\lim_{x\to 0} \frac{x-14}{x-7}$
4. Calculate the limit: $\lim_{x\to+\infty} 2e^{-x}$
5. Calculate the limit: $\lim_{x\to\infty} \frac{5-8\,x^2}{7-9x-4x^2}$

6. Calculate the limit: $\lim_{x \to -\infty} \frac{4-8x^2}{2x+6}$

7. Calculate the limit: $\lim_{x \to \infty} \frac{3-2x^2}{2-4x-x^2+x^5}$

8. The amount of drug (in milligrams) in the blood after an IV tube is inserted is given by $m(t) = 40 * 0.62^t$, where t is the number of hours after it was injected.
   Compute $\lim_{t \to +\infty} m(t)$, then interpret the result.

9. The population of a colony of squirrels is given by $p(t) = \frac{1500}{3+2e^{-0.1t}}$, where t is the time in years since 1975.
   Compute $\lim_{t \to +\infty} p(t)$, then interpret the result.

10. The following models approximate the popularity of Twitter and LinkedIn among social media sites from 2008 to 2013, as rated by statecounter.com:

    Twitter $W(t) = 0.33t^2 - 2t + 8.7$ percentage points

    LinkedIn $L(t) = 0.04t^2 - 0.26t + 0.67$ percentage points

    where t is the number of years since the start of 2008.

    a) Compute $\lim_{t \to +\infty} W(t)$, then interpret the result.
    b) Compute $\lim_{t \to +\infty} \frac{W(t)}{L(t)}$, then interpret the result.

Rates of Change (sections 10.4, 10.5)

1. Let $f(x) = x^3 + 2$. Find the average rate of change of f over the interval [1,4].

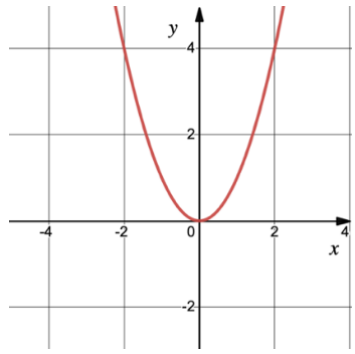
2. Calculate the average rate of change of the given function over the interval [2,6] and specify the unit of measurements.

x (days)	1	2	3	4	5	6	7	8	9
$f(x)$ (dollars)	6	20	-3	8	4.6	12	1.5	4.9	-9

3. Suppose the following table shows U.S. daily oil imports from a certain country, for 1991−1999 (t = 1 represents the start of 1991). Use the data in the table to compute the average rate of change of $I(t)$ over the period 1991−1999 and interpret the meaning of the result.

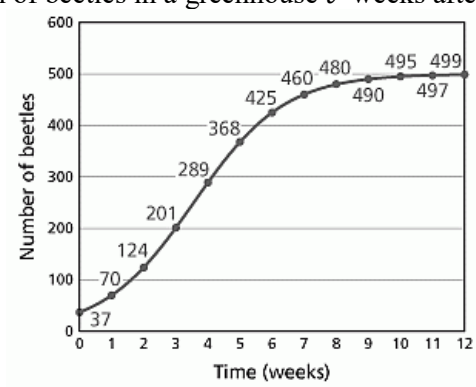
t **(year since 1990)**	1	2	3	4	5	6	7	8	9
***I*(*t*)** **(million barrels)**	5.6	2.5	3.8	9.4	8.2	3.7	9.6	7.2	8.1

4. Consider the following graph of f where f is defined over $(-\infty, \infty)$:



a) Interpret the sign of f'
b) Graph f'

5. The graph below shows the population of beetles in a greenhouse t weeks after the season's flowers were planted.



a) Calculate the average rate of change over the interval [1,10].

b) **T** for True or **F** for False for each statement.

A) During weeks [4,12] the instantaneous rate of change of the population is increasing
B) During weeks [4,12] the instantaneous rate of change of the population is decreasing
C) The average rate of change of the population on [4,12] is less than the instantaneous rate of change of the population at $t = 4$
D) The average rate of change of the population on [0,12] is greater than the instantaneous rate of change of the population at $t = 2$
E) The instantaneous rate of change of the population first increased then decreased
F) The instantaneous rate of change of the population at $t = 2$ is less than at $t = 11$
G) The instantaneous rate of change of the population at $t = 11$ is approximately 0
H) The instantaneous rate of change of the population at $t = 6$ is negative
I) In this graph, the slope of the tangent line at $t = 4$ is the greatest

6. Based on data from 1982 to 2017, the number of students taking the AP Calculus exam may be modeled by the function $S(t)$, where t is the number of years since 1982.
Interpret the meaning of $S(16) = 156{,}682$ and $S'(16) = 9{,}644$.

   A. In 1998 the number of students who took the AP Calculus exam is 156,682 and this number is increasing at a rate of 9,644 student per year.
   B. In 1998 the number of students who took the AP Calculus exam is 156,682 and this number is increasing at a rate of 9,644 student in every 16 years after 1982.
   C. In 1998 the number of students who took the AP Calculus exam is 156,682 and the test score is increasing at a rate of 9,644 points in every 16 years after 1982.
   D. In 1998 the number of students who took the AP Calculus exam is 156,682 and between 1982 and 2017 the number students who took the AP Calculus exam increased by an average of 9,644 students per year.
   E. None of these

Differentiation (Power Rule, Product Rule, Quotient Rule, Contant Multiple Rule) **(sections 11.1, 11.3)**

1. Find the derivative of each function using differentiation rules.
   a) $f(x) = 4x^2 + 2$
   b) $f(x) = 8 - 2x$
   c) $f(x) = 2x^2 + 6x - 7$
   d) $f(x) = -\frac{9}{4x^4}$
   e) $f(x) = \frac{8}{\sqrt{x}} - \frac{4}{x}$
   f) $g(x) = 3x^2 + 8x - \frac{2}{x^6} - 5x^{2.3} + 7.95$
   g) $h(x) = 8x(3x^2 - 2x + 9)$
   h) $k(x) = \frac{3x+2}{4x}$
   i) $m(x) = (-5x^2 + 7x^{-1})(x^{-2} - 7)$
   j) $v(x) = \frac{2x-1}{4x+8}$
   k) $p(x) = \frac{x^2+1}{x^3-2x}$
2. Suppose $f(x)$ is a differentiable function such as $f(9) = -5$ and $f'(9) = 10$. Let $g(x)$ be a function that is defined as $g(x) = (x^2)f(x)$. Compute $g'(9)$.

3. Suppose $f(x)$ is a differentiable function such that $f(4) = 16$ and $f'(4) = -8$. Let $g(x)$ be a function that is defined as $g(x) = (\sqrt{x})f(x)$. Compute $g'(4)$.

4. Suppose $f(x)$ and $g(x)$ are two differentiable functions such that: $f(2) = -2, f'(2) = 11, g\ (2) = 3$, and $g'(2) = -3$.
   a) Let $h(x)$ be a function that is defined as $h(x) = g(x)f(x)$. Compute $h'(2)$.
   b) Let $k(x)$ be a function that is defined as $k(x) = \frac{f(x)}{g(x)}$ $(where\ g(x) \neq 0)$. Compute $k'(2)$.

Applications to Derivatives and Rates of Change

Tangent Lines

1. Find the equation of the tangent line to the graph of $f(x) = 3x^2 - 4x + 6$ at $x = 2$.

2. Find the equation of the line tangent to the graph of $f(x) = x^2 - 2$ at $(3, 7)$.

3. Find the slope of the tangent line to the graph of $f(x) = 5x^3 + 2x + 4$ at $x = 1$

Average Velocity and Instantaneous Velocity

1. Assume that the distance, s (in meters), traveled by a car moving in a straight line is given by the function $s(t) = t^2 - 3t + 5$, where t is measured in seconds.
   a) Find the *average* velocity of the car during the time period from $t = 1$ to $t = 4$.
   b) Find the *instantaneous* velocity of the car at time $t = 3$ seconds.

Marginal Analysis (section 11.2)

1. Assume that your monthly profit (in dollars) from selling homemade cookies is given by $P(x) = 8x - 2\sqrt{x}$, where *x* is the number of boxes of cookies you sell in a month.
   a) Determine the marginal profit function, *MP(x)*.

   b) Determine value of marginal profit if you are selling 25 boxes of cookies per month and interpret.

2. Find the marginal cost, the marginal revenue, and the marginal profit functions, where the cost and revenue functions, respectively, are $C(x) = 8x^2$, and $R(x) = 4x^3 + 2x + 10$.

3. Assume that your monthly profit (in dollars) from selling books is given by $P(x) = 5x^2 + 6x - 2$, where *x* is the number of books you sell in a month. If you are currently selling $x = 50$ books per month, find your profit and your marginal profit.

4. Your monthly cost (in dollars) from selling homemade candles is given by $C(x) = 150 + 0.1x + 0.002x^2$, where *x* is the number of candles you sell in a month. The revenue from selling x candles is $R(x) = 7x$.

   a) Write a function $P(x)$ for your monthly *profit* of producing and selling *x* candles.
   b) Calculate $P(100)$. Include units.
   c) Write a function for your *marginal profit*.
   d) Calculate your marginal profit if you produce and sell 100 candles. Include units and interpret your answer.

Chain Rule (section 11.4)

1. Find the derivative y' of each function.
   (a) $y = 0.4(3x^2 + 2x - 8)^5$
   (b) $y = \sqrt{x^3 - 50x}$
   (c) $y = (6x + 1)^{4/3}$
   (d) $y = \frac{25}{(x^2+x+2)^4}$

Derivative of Logarithmic and Exponential Functions (section 11.5)

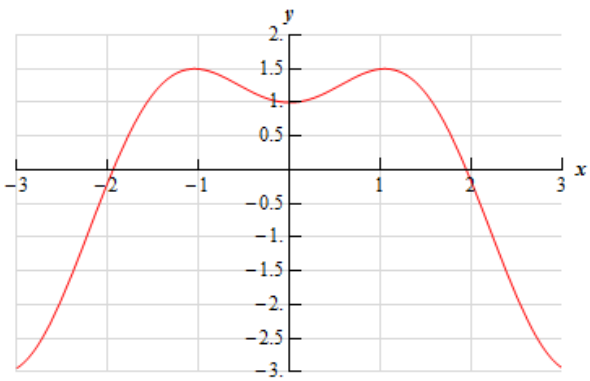
1. Find the derivative y' of each function.
   (a) $y = 9\ln(2x)$
   (b) $y = \ln(5x^3 + x^2 + 4)$
   (c) $y = \ln|x^3 - 8x|$
   (d) $y = x - x\ln x$
   (e) $y = 4e^{x^5-3x}$
   (f) $y = (x^2 - 2x)e^{2x+3}$
   (g) $y = e^{5/x}$
   (h) $y = 8e^{-2x}$
   (i) $y = 5e^{3x^3+2x}$

Implicit Differentiation (section 11.6)

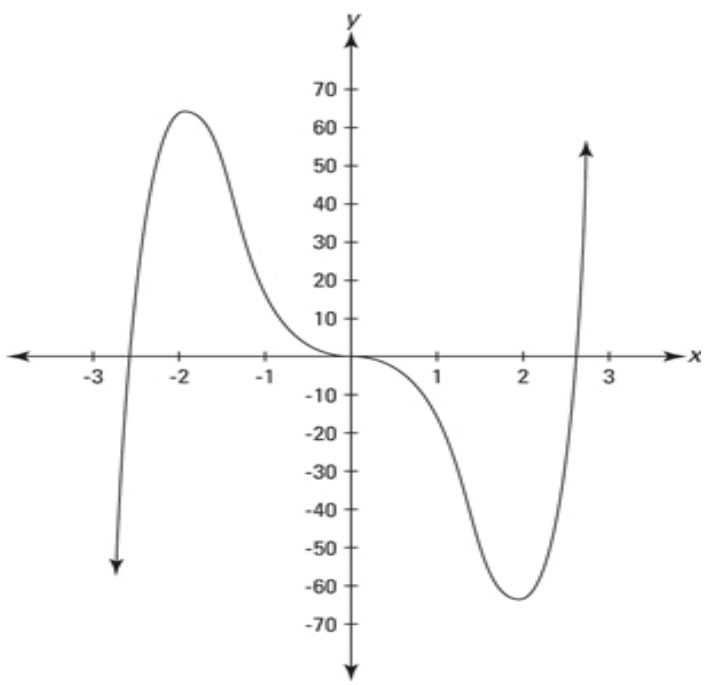
1. Find the derivative $\frac{dy}{dx}$.
   (a) $x^3 - y^3 + y = 3$
   (b) $6x^2y - 15x = y^2$
   (c) $xe^y - e^x = 0$

Maxima and Minima (section 12.1)

1. The function of a function f on $[-3, 3]$ is given below.
   (a) f has a relative minimum at $x =$
   (b) f has an absolute maximum at $x =$
   (c) f has an absolute minimum at $x =$
   (d) f' is zero at $x =$
   (e) f' is positive on interval(s):
   (f) f' is negative on interval(s):



2. The graph of a function f is given below.
   (a) f' is zero at $x =$
   (b) f has a relative max at $x =$ _____ and has a relative min at $x =$



3. Find all critical points of the function. Use the First Derivative Test to determine whether f has a relative minimum, a relative maximum or neither at the critical point.

$$f(x) = x - 2\ln x\,, x > 0$$

4. Consider the function $f(x) = 8x^3 - 24x + 12$.
   (a) Find all critical points of f.
   (b) Find the absolute extrema of f on interval $[-3, 2]$.

5. Consider the function $f(x) = 3x^4 + 4x^3 - 12x^2$ defined on $[-3, 0.5]$.
   (a) Find all critical point(s) of f. Write your answer as a list of ordered pairs.
   (b) Find the coordinates of the endpoints of f. Write your answer as a list of ordered pairs.
   (c) Find the location of the absolute and relative extrema of f on the interval $[-3, 0.5]$.

6. Consider the function $g(x) = (x-2)^{2/3}$,
   (a) Find any critical points of g.
   (b) Find the absolute max and absolution min of g over $[0, 5]$.

7. For the function $k(x) = 4x^3 - 24x^2 + 36x - 20$,
   (a) Find any critical points of k.
   (b) For what x values is the function k increasing? decreasing?
   (c) Find any relative and absolute extrema of k on $[-1, 5]$.

8. For the function $h(x) = e^x - x$ defined on $[-2, 3]$,
   (a) Find any critical points of h.
   (b) For what x values is the function h increasing? decreasing?
   (c) Find any relative and absolute extrema of h.

9. Suppose $f(x)$ is continuous on $(-\infty, \infty)$ and f has two critical points at $x = -1$ and $x = 2$. If we know $f'(-2) < 0$, $f'(0) > 0$, and $f'(3) < 0$, determine whether each statement is True or False.
   (a) **T** or **F** f has a relative minimum at $x = -1$ because f is decreasing on the left side of $x = -1$ and increasing on the right side of $x = -1$.
   (b) **T** or **F** f has a relative maximum at $x = 2$ because f' is positive on the left side of $x = 2$ and negative on the right side of $x = 2$.
   (c) **T** or **F** f is decreasing on the interval $[-1, 2]$.
   (d) **T** or **F** f is decreasing on the interval $(2, \infty)$.

10.

Suppose $f(x)$ is continuous on $(-\infty, \infty)$ and f has two critical points at $x = 0$ and $x = 4$. If $f'(-1) < 0$, $f'(1) < 0$, and $f'(5) > 0$, then

   (a) f has _____ (relative minimum/relative minimum/no relative extrema) at $x = 0$.
   (b) f has _____ (relative minimum/relative minimum/no relative extrema) at $x = 4$.
   (c) f is increasing on interval(s):
   (d) f is decreasing on interval(s):

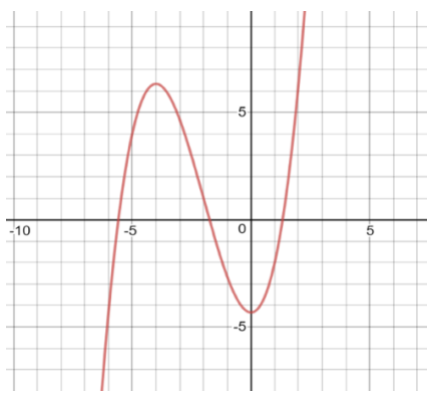
Optimization: Applications to Maximum and Minimum (Section 12.2)

1. You are running a business selling homemade bread. Your weekly revenue from the sale of q loaves bread is $R(q) = 68q - 0.1q^2$ dollars, and the weekly cost of making q loaves of bread is $C(q) = 23 + 20q$.
   (a) Find the weekly profit function $P(q)$.
   (b) Find the production level q that maximizes the weekly profit.
   (c) Find the maximum profit.
2. Suppose $C(x) = 0.02x^2 + 2x + 4000$ is the total cost for a company to produce x units of a certain product. Find the production level x that minimizes the average cost $\bar{C}(x) = \frac{C(x)}{x}$.
3. I would like to create a rectangular orchid garden that abuts my house so that the house itself forms the northern boundary. The fencing for the southern boundary costs \$3 per foot, and the fencing for the east and west sides costs \$5 per foot. If I have a budget of \$120 for the project, what are the dimensions of the garden with the *largest area* I can enclose?
4. I need to create a rectangular vegetable patch with an area of exactly 162 square feet. The fencing for the east and west sides costs \$4 per foot, and the fencing for the north and south sides costs only \$2 per foot. What are the dimensions of the vegetable patch with the *least expensive* fence?
5. Worldwide annual sale of a product in 2013-2017 were projected to be approximately $q = -10p + 4220$ million units at a selling price of p dollars per unit. What selling price would have resulted in the largest projected annual revenue? What would be resulting revenue?

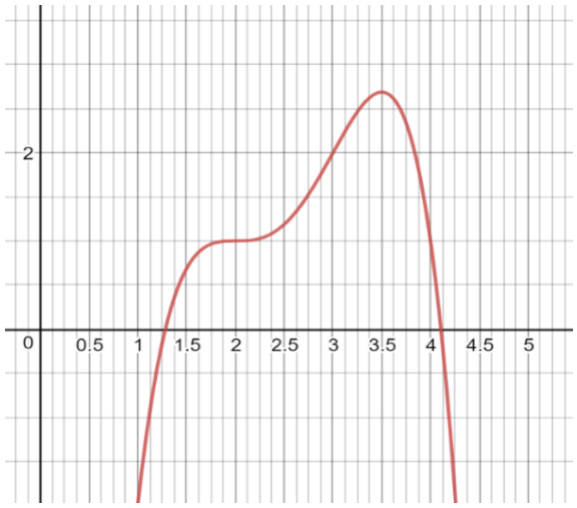
Higher-order Derivatives, Acceleration and Concavity (Section 12.3)

1. Find the second derivative y'' for each function.
   (a) $y = 2e^{2x-5}$
   (b) $y = \frac{7}{x} - 5\ln x$

2. The graph of a function $y = f(x)$ is given below.
   (a) $f'(-4) = f'(0) =$
   (b) Is $f''(-4)$ is positive or negative? Is $f''(0)$ is positive or negative?
   (c) If f has a point of inflection at $x = -2$, then $f''(-2) =$
   (d) f is concave_____(up/down) on interval $(-\infty, -2)$, and concave_____(up/down) on interval $(-2, \infty)$.



3. The graph of a function $f(x)$ is given. Fill in the blank.
   (a) The graph is concave_____(up/down) on interval $(-\infty, 2)$, concave_____(up/down) on interval $(2, 3)$, and concave_____(up/down) on interval $(3, \infty)$.
   (b) The second derivative f'' is positive on: _____and negative on: _____.
   (c) List the points of inflection: $x =$
   (d) Does f have any relative extrema?
   (e) Does f have any absolute extrema?



4. Suppose the position of a particle moving on a straight line is $s(t) = \sqrt{t} + 4t^2$. Find the particle's acceleration as a function of time t.

5. Let $s(t) = 4e^t - 8t^2 + 3$ be the position function of a particle moving in a straight line, where s is measured in feet and t is measured in seconds. Find its acceleration when $t = \ln 6$ seconds.

Related Rates (Section 12.5)

1. The radius of a circular puddle is growing at a rate of 15 cm/sec.
   (a) How fast is its area growing at the instant when the radius is 30 cm?
   (b) How fast is the area growing when the area is 81 square centimeters?
2. A rather flimsy spherical balloon is designed to pop at the instant its radius has reached 6 cm. Assuming the balloon is filled with helium at a rate of 13 cubic centimeters per second, calculate how fast the radius is growing at the instant it pops.
3. The average cost for the weekly manufacture of retro portable CD player is given by

$$\bar{C}(x) = 120{,}000x^{-1} + 20 + 0.0004x \text{ dollars per player,}$$

   where x is the number of CD players manufactured that week. Weekly production is currently 4,000 players and is increasing at a rate of 400 players per week. What is happening to the average cost? Fill in the blank.

   The average cost is _____increasing/decreasing at a rate of _____dollars per player per week.

Elasticity (Section 12.6) (Optional, this lesson will not be covered on the exam)

1. Suppose the elasticity of demand is 3.2, when the price of a product is \$25. This means the demand is going up/down by _____% for 1% increase in the price. A small increase in price will result in a increase/decrease in the revenue.

2. Suppose the elasticity of demand is 0.65, when the price of a product is \$500. This means the demand will go up/down by _____% for 1% increase in the price. A small increase in price will cause the revenue to increase/decrease.

3. The weekly sales of some backpacks is given by $q = 1080 - 18p$, where the q represents the quantity of backpacks sold at price p.
   (a) Find the elasticity of demand at the price of \$20. Interpret your answer.
   (b) Is the demand at the price \$20 elastic, inelastic, or unit elastic? Should the price be raised or lowered from \$20 to increase the revenue?
   (c) What price will maximize the revenue?
   (d) What is the maximum weekly revenue?

4. Suppose the demand function is $q = -2p^2 + 33p$, where q represents the quantity sold at price p.
   (a) Find the price elasticity of demand $E(p)$.
   (b) Find the elasticity when $p = \$15$. If the price increase by 1%, the demand will drop by how much? Should the price be lowered or raised from \$15 to increase the revenue?

Indefinite Integral (Section 13.1)

1. Find the indefinite integrals.
   (a) $\int 2x^4 - 4x^{-2} + 5x^{-5} + 3\ dx$
   (b) $\int \frac{7}{x} + \frac{1}{3x^7} dx$
   (c) $\int \frac{2}{x^2} - 5\sqrt{x}\ dx$
   (d) $\int e^x - x^{-0.3}\ dx$
   (e) $\int (x+3)(x-2) dx$
   (f) $\int \frac{x^2+5x-2}{x} dx$
2. Find $f(x)$ if $f(0) = -1$ and the derivative $f'(x) = 9e^x + 9$.

3. The velocity of a particle moving in a straight line is $v(t) = t^2 + 6$. Find the expression for the position, $s(t)$, of the particle at time t, if $s(3) = 0$.
4. Suppose the function $C(x)$ gives the total cost (in dollars) of producing x units of a certain product. The marginal cost of producing the xth unit is $C'(x) = 0.5x + \frac{1}{x}$. If the cost to produce the first unit is 5 dollars, find the cost function $C(x)$.

Substitution (Section 13.2)

1. Use integration by substitution to find the integrals.
   (a) $\int 16e^{-3x}\,dx$ (can also use short-cut formula)
   (b) $\int (5x-2)^3 dx$ (can also use short-cut formula)
   (c) $\int \frac{1}{2x-5} dx$ (can also use short-cut formula)
   (d) $\int 4xe^{x^2-3} dx$
   (e) $\int x(x^2+1)^{10}\,dx$
   (f) $\int 15x\sqrt{-x^2+7}\,dx$
   (g) $\int (3x^2+1)(x^3+x-2)^9 dx$
   (h) $\int \frac{3\ \ln x}{x} dx$

Fundamental Theorem of Calculus; Definite Integral; Left Riemann Sum (Sections 13.3, 13.4)

1. Evaluate the definite integrals by using the fundamental Theorem of Calculus. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.
   (a) $\int_0^1 (6x^5+15x^4-9x^2+1)\,dx$
   (b) $\int_2^7 \left(x+\frac{5}{x}\right) dx$
   (c) $\int_1^{10} \frac{1}{x^2} dx$
   (d) $\int_0^6 e^{-x+6}\,dx$
   (e) $\int_{-1}^1 5e^{3x} dx$
   (f) $\int_{e^3}^{e^5} \frac{2}{x} dx$
   (g) $\int_{\ln 3}^{\ln 5} e^{2x} dx$

2. Assume that b is a positive number, solve the following equation for b.

   $\int_2^b (2x-4)\,dx = 9$

3. Calculate the left Riemann sum for the function $f(x) = 3x^2+2x-3$ over the interval $[1,3]$, with $n=5$.

4. Use a left Riemann sum to estimate the definite integral with $n=4$ subintervals.

$$\int_2^3 \frac{1}{1+2x} dx$$

Applications of Definite Integrals (Section 13.4)

1. A particle moves in a straight line with velocity $v(t) = -t^2 + 8$ meters per second, where t is time in seconds. Find the displacement of the particle between $t = 2$ and $t = 6$ seconds. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.

2. The marginal revenue of the xth box of flash cards sold is $500e^{-0.001x}$ dollars. Find the revenue generated by selling box 101 through 5,000. Show all your work step by step. **Give the exact value first; then round your answer to 2 decimal places.** Only use your calculator to round your answer.

3. Since YouTube first became available to the public in mid-2005, the rate at which video has been uploaded to this site can be approximated by $f(t) = 1.1t^2 - 2.6t + 2.3$ million hours of videos per year ($0 \le t \le 9$), where t is time in years since June 2005. Use a definite integral to estimate the total number of hours of video uploaded from June 2007 to June 2010. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.

4. Calculate the area of the region bounded by $y = \sqrt{x}$, the x-axis, and the lines $x = 0$ and $x = 16$. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.

Area between Curves (Section 14.2)

1. Find the area of the region enclosed by the curves of $y = -x^2 + 6x + 2$ and $y = 2x^2 + 9x - 4$. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.

2. Find the area of the region enclosed by the curves of $f(x) = x^2 - x + 5$ and $g(x) = x + 8$. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.

3. Find the area of the region between $y = x^2$ and $y = -1$ from $x = -1$ and $x = 1$. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.
4. Which of the following calculates the area of the region(s) between the curves $y = x^2$ and $y = 1$ from $x = -1$ to $x = 2$?

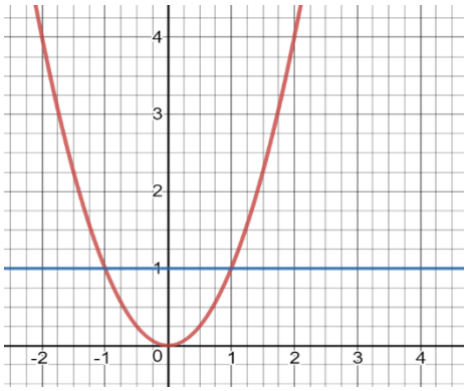
   A. $\int_{-1}^{2}(x^2 - 1)dx$

   B. $\int_{-1}^{2}(1 - x^2)dx$

   C. $\int_{-1}^{1}(1 - x^2)dx + \int_{1}^{2}(x^2 - 1)dx$

   D. $\int_{-1}^{1}(x^2 - 1)dx + \int_{1}^{2}(1 - x^2)dx$

   E. None of the above.



Average Value (Section 14.3)

1. Find the average value of $f(x) = 6e^{0.5x}$ over the interval $[-1, 3]$. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.

2. Find the average of the function $f(x) = x^3 - x$ over the interval $[0, 2]$. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.

3. Find the average value of the function $f(x) = 6x^2 - 4x + 7$ over the interval $[-2, 2]$. Show all your work step by step. **Give the exact value.** You may use your calculator to check your final answer only.

Consumer's Surplus and Producer's Surplus (section 14.4)

1. Your shop can sell 50 "I love calculus" t-shirt at \$20 each per day. You decided to drop the price \$1.50 per shirt and this results in 3 more t-shirts sold per day.
   a. Write a linear function for the unit price p of the t-shirt sold daily as a function of q (demand), the number t-shirts can be sold at unit price p.

   b. Calculate the consumers' surplus when the unit price is $\bar{p}$=15 dollars per shirt using the demand equation found in part (a). Consumers' Surplus is defined as $\int_0^{\bar{q}} (D(q) - \bar{p})dq$.

2. Calculate the producers' surplus for the supply equation at the indicated unit price $\bar{p} = \$\,160$ (Round your answer to the nearest cent.)
   Producers" Surplus is defined a $\int_0^{\bar{q}} (\bar{p} - S(q))dq$ where $p = 130 + e^{0.01q}$, $\bar{p} = 160$.

3. A company finds that the demand for their new product is given by $p = 13 - q^{\frac{1}{4}}$, where p is the price per item and q is the number of items that can be sold per week at unit price p. The company is prepared to sell $q = \left(\frac{p-4}{2}\right)^4$ items per week at a unit price p. Find the equilibrium price $\bar{p}$ and the consumers' and producers' surpluses at the equilibrium price. What is the total social gain at the equilibrium price?

4. Given that x is number of items, the demand function is $d(x) = 200 - 0.2x$, and the supply function is $s(x) = 0.3x$. Show all your work step by step. You may use your calculator to check your final answer only.
   a) Find the equilibrium quantity.
   b) Find the consumers' surplus and producers' surplus at the equilibrium quantity.

5. Given that x is number of items, the demand function is $d(x) = 270.4 - 0.1x^2$, and the supply function is $s(x) = 0.3x^2$. Show all your work step by step. You may use your calculator to check your final answer only.
   a) Find the equilibrium quantity.
   b) Find the consumers' surplus and producers' surplus at the equilibrium quantity. (Round your answer to one decimal place.)

6. Given that x is number of items, the demand function is $d(x) = \frac{2304}{\sqrt{x}}$, and the supply function is $s(x) = 9\sqrt{x}$. Show all your work step by step. You may use your calculator to check your final answer only.

   a) Find the equilibrium quantity.
   b) Find the consumers' surplus and producers' surplus at the equilibrium quantity.

7. Your video store has the exponential demand of equation $p = 15e^{-0.01q}$, where q represents daily sales of used DVD's and p represents daily price you charge per DVD. Calculate the daily Consumer's Surplus if you sell used DVDs at \$5 each. (Round your answer to the nearest cent.) Show all your work step by step. You may use your calculator to check your final answer only.

8. Calculate the Producer's Surplus for the supply equation $p = 13 + 2q$ at the unit price $\bar{p} = 29$. Show all your work step by step. You may use your calculator to check your final answer only.

9. Calculate the Producer's Surplus for the supply equation $p = 7 + 2q^{\frac{1}{3}}$ at the unit price $\bar{p} = 14$. (Round your answer to the nearest cent.) Show all your work step by step. You may use your calculator to check your final answer only.

Improper Integral (Section 14.5)

1. Determine whether the given improper integral converges or diverges.

   - If convergent, evaluate the integral and give the numerical answer.
   - If divergent, indicate if the integral diverges to positive infinity or negative infinity.

   a) $\int_1^{\infty} \frac{8}{x^2}\,dx$
   b) $\int_{-2}^{\infty} e^{-3x}\,dx$
   c) $\int_1^{\infty} \frac{3}{x}\,dx$
   d) $\int_1^{\infty} \frac{6}{x^2}\,dx$
   e) $\int_{-\infty}^{-2} \frac{3}{x^2}\,dx$
   f) $\int_1^{\infty} x\,dx$
   g) $\int_0^{3} \frac{1}{x^{1.1}}\,dx$
   h) $\int_0^{3} \frac{1}{x^{0.1}}\,dx$
   i) $\int_{-\infty}^{-1} \frac{1}{x^{\frac{1}{3}}}\,dx$
   j) $\int_4^{\infty} \frac{1}{x^4}\,dx$
   k) $\int_0^{\infty} e^{-x}\,dx$
   l) $\int_{-\infty}^{2} e^{x}\,dx$

m) $\int_4^{\infty} e^{-2x}\, dx$

n) $\int_3^{\infty} x^2\, dx$

o) $\int_{-\infty}^{0} e^{2x}\, dx$