## Answers: MAT 210 Exam 1 Review Questions

## Limits (sections 10.1, 10.3)

1. a. 22
b. $\frac{6}{13}$
c. 3
d. $\frac{1-2 a}{4}$
e. $\frac{a}{3}$
f. $\frac{3 a-3}{9 b-1}$
2. 

a. $\lim _{x \rightarrow \infty} \frac{3 x^{3}-3}{-13 x^{3}-4 x^{2}-2}=\lim _{x \rightarrow \infty} \frac{3-\frac{3}{x^{3}}}{-13-\frac{4}{x}-\frac{2}{x^{3}}}=\frac{3}{-13}=-\frac{3}{13}$, as $x \rightarrow \infty$ the read terms aproaches 0 .

As $x \rightarrow \infty$, the numerator approaches $\infty$ and the denominator approaches $-\infty$. This limit expression has a $\frac{\infty}{-\infty}$ type of indeterminate form (Again this is not a number and not the answer.) Divide each term of the rational function by $x^{n}$, where $n$ is the largest power of $x$ in the expression. In this problem divide by $x^{3}$, where 3 is the largest power of $x$. Take the limit as $x \rightarrow \infty$, each $\frac{\text { non-zero\# }}{x^{n}}$ term approaches 0 . Thus, for large $x$ values (as $x \rightarrow \infty$ or as $x \rightarrow$ $-\infty$ ), all terms of a rational function are negligible except the leading terms since the leading terms in both, in the numerator and denominator much larger compared to all the other terms. Therefore $\lim _{x \rightarrow \infty} \frac{3 x^{3}-3}{-13 x^{3}-4 x^{2}-2}=$ $\lim _{x \rightarrow \infty} \frac{3 x^{3}}{-13 x^{3}}=-\frac{3}{13}$. In general, if the degree of the numerator of a rational function is the same as the degree of the denominator then the limit will be the ratio of the leading terms as $x \rightarrow \infty$ or as $x \rightarrow-\infty$.
b. $\quad \lim _{x \rightarrow \infty} \frac{7 x^{2}-5}{19 x^{3}-3 x-7}=\lim _{x \rightarrow \infty} \frac{7 x^{2}}{19 x^{3}}=\lim _{x \rightarrow \infty} \frac{7}{19 x}=0$.

As $x \rightarrow \infty$, the numerator approaches $\infty$ and the denominator approaches $\infty$. This limit expression has a $\frac{\infty}{\infty}$ type of indeterminate form. Again this is not a number and not the answer. In general, if the degree of the numerator of a rational function is smaller than the degree of the denominator then the limit is 0 as $x \rightarrow \infty$ or as $x \rightarrow-\infty$.
c. $\quad \lim _{x \rightarrow \infty} \frac{-9 x^{5}+5}{12 x^{3}-3 x-7}=\lim _{x \rightarrow \infty} \frac{-9 x^{5}}{12 x^{3}}=\lim _{x \rightarrow \infty} \frac{-9 x^{2}}{12}=-\infty$.

As $x \rightarrow \infty$, the numerator approaches $-\infty$ and the denominator approaches $\infty$. This limit expression has a $\frac{-\infty}{\infty}$ type of indeterminate form. In general, if the degree of the numerator of a rational function is greater than the degree of the denominator then the limit does not exist. The limit is either $\infty$ or $-\infty$ depending on the sign of the leading coefficients.
All the limit expressions discussed above have $\frac{0}{0}$ or $\pm \frac{\infty}{\infty}$ indeterminate form, so there is no general rule to find the limit, each individual problem requires further analysis to determine the limit.
d. $\quad \lim _{x \rightarrow \infty} \frac{-3}{11 x^{3}}=0$

As $x \rightarrow \infty$, the denominator approaches $\infty$. This limit expression has a $\frac{\text { non-zero\# }}{\infty}$ type of determinate form, thus the limit is 0 . A non-zero number divided by a large number results a number a number that is close to 0 . As the dividend increases without bound the value of the fraction is getting arbitrarily close to 0 .
e. $\lim _{x \rightarrow \infty} \frac{-3 x^{3}}{11}=-\infty$

As $x \rightarrow \infty$, the numerator approaches $-\infty$. This limit expression has a $\frac{-\infty}{\text { non-zero \# }}$ type of determinate form, thus the limit is $-\infty$.
f. $\frac{a}{b}$
g. $-\infty$
h. 0
i. $\frac{a}{2 b}$
3. 2
4. 0
5. 2
6. $\infty$
7. 0
8. $\lim _{t \rightarrow+\infty} m(t)=0$. In the long term, the amount of drug in the blood will completely disappear.
9. $\lim _{t \rightarrow+\infty} p(t)=500$. The population of squirrels will approach 500 as the time increases (in the long run).
10.
a) $\lim _{t \rightarrow+\infty} W(t)=\infty$. In the long term, the popularity of Twitter among social media sites will increase without bound.
b) $\lim _{t \rightarrow+\infty} \frac{W(t)}{L(t)}=8.25$. In the long term, the popularity of Twitter will increase by 8.25 more than LinkedIn.

Note: A percentage can't rise beyond 100, so extrapolating the models to obtain long term predictions gives meaningless results.

## Rates of Change (sections 10.4, 10.5)

1. 21
2.     - 2 dollars/day
3. 0.3125 million barrels per year.
U.S. daily oil imports from a certain country increased by an average rate of 0.3125 million barrels per year over the period 1991 to 1999.
4. 

a) The sign of $f^{\prime}$ is negative on the interval $(-\infty, 0)$, it is 0 at $x=0$, then positive on the interval $(0, \infty)$
b) $f^{\prime}$ is always increasing

5.
a) $425 / 9$ beetles per week
b)
A) F
B) T
C) T
D) F
E) T
F) F
G) T
H) F
I) T
6.
A. In 1998 the number of students who took the AP Calculus exam is 156,682 and this number is increasing at a rate of 9,644 student per year.

## Differentiation (Power Rule, Product Rule, Quotient Rule, Contant Multiple Rule) (sections 11.1, 11.3)

1. 

a) $f^{\prime}(x)=8 x$
b) $f^{\prime}(x)=-2$
c) $f^{\prime}(x)=4 x+6$
d) $f^{\prime}(x)=\frac{9}{x^{5}}$
e) $f^{\prime}(x)=-4 x^{-3 / 2}+4 x^{-2}$
f) $g^{\prime}(x)=6 x+8+12 x^{-7}-11.5 x^{1.3}$
g) $h^{\prime}(x)=72 x^{2}-32 x+72$
h) $k^{\prime}(x)=-0.5 x^{-2}$
i) $\quad m^{\prime}(x)=70 x-21 x^{-4}+49 x^{-2}$
j) $\quad v^{\prime}(x)=\frac{20}{(4 x+8)^{2}}$
k) $p^{\prime}(x)=\frac{2 x \cdot\left(x^{3}-2 x\right)-\left(x^{2}+1\right) \cdot\left(3 x^{2}-2\right)}{\left(x^{3}-2 x\right)^{2}}=\frac{-x^{4}-5 x^{2}+2}{\left(x^{3}-2 x\right)^{2}}$
2. 720
3. -12
$\begin{array}{ll}\text { 4. a) } 39 & \text { b) } 3\end{array}$

## Applications to Derivatives and Rates of Change

## Tangent Lines

1. $y=8 x-6$
2. $y=6 x-11$
3. $f^{\prime}(1)=17$

## Average Velocity and Instantaneous Velocity

1. 

a) $2 \mathrm{~m} / \mathrm{s}$
b) $3 \mathrm{~m} / \mathrm{s}$

## Marginal Analysis (section 11.2)

1. 

a) $M P(x)=8-\frac{1}{\sqrt{x}}$
b) 7.8 dollars per box. After 25 boxes of cookies have been sold, the total profit will increase by about 7.8 dollars per additional box sold, or the profit from selling the $26^{\text {th }}$ box is about 7.8 dollars.
2. $M C(x)=16 x \quad M R(x)=12 x^{2}+2 \quad M P(x)=12 x^{2}-16 x+2$
3. profit $=12798$ dollars

Marginal profit $=506$ dollars per book
4.
a) $P(x)=-150+6.9 x-0.002 x^{2}$
b) 520 dollars
c) $M P(x)=6.9-0.004 x$
d) 6.5 dollars per candle. The profit from selling the $101^{\text {st }}$ candle is about 6.5 dollars. Or the total profit will increase by 6.5 dollars per candle sold, after 100 candles are sold.

