MAT 171 – Final Exam Review Problems

Formulas

The following formulas will be provided at the beginning of the Final Exam.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

$$A = A_0 e^{kt}$$

$$\cos^2(x) + \sin^2(x) = 1 \qquad 1 + \tan^2(x) = \sec^2(x) \qquad \cot^2(x) + 1 = \csc^2(x)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \begin{cases} \cos^2(x) - \sin^2(x) \\ 2\cos^2(x) - 1 \\ 1 - 2\sin^2(x) \end{cases}$$

$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$

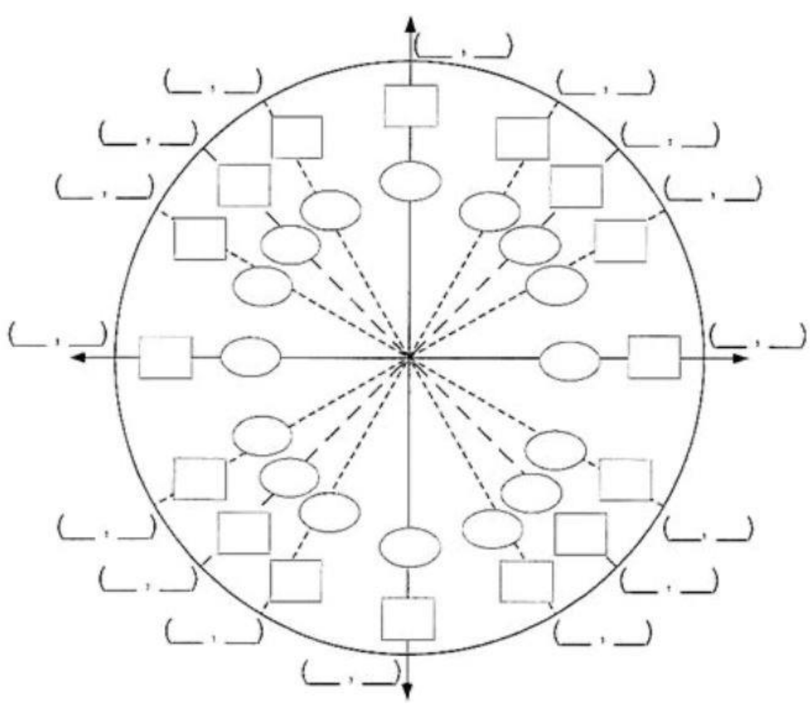
$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}$$

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



Review Problems (in addition to Exam 1 and Exam 2 review problems)

1.2 – Find the domain of each function

(a) $f(x) = \frac{1-x}{x^2-9}$

(b) $f(x) = \frac{x+1}{3x-2}$

(c) $f(x) = \log(2x+1)$

(d) $f(x) = \sqrt{8-2x}$

1.3 – Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}, h \neq 0$

(a) $f(x) = -x^2 + 5x + 9$

(b) $f(x) = \frac{1}{3x}$

(c) $f(x) = 3x^2 + 4x - 8$

1.6 – Find the function $g(x)$ after applying the following transformations to x^2:

reflect about the x-axis, vertically stretch by a factor of 2, horizontally stretch by a factor of 4, shift left 5 units, shift up 3 units

1.7 – Find compositions

(a) Find $(f \circ g)(x)$ and $(g \circ f)(x)$ where $f(x) = x^2 - x + 4$ and $g(x) = 2x - 3$

(b) Find $(g \circ f)(x)$ where $f(x) = e^{2x} - 1$ and $g(x) = \ln(x + 1)$

1.8 – Find inverse functions

(a) $f(x) = \frac{4x}{3+x}$

(b) $f(x) = x^3 - 10$

2.2 – Quadratic function application

An astronaut on the moon throws a baseball upward.
The height of the ball is given approximately by the function $h(t) = -2.7t^2 + 30t + 6.5$ feet, where t is the time, in seconds, after the ball was thrown.
When does the baseball reach its maximum height?
What is the maximum height of the baseball?
When does the baseball reach the ground?

2.3-2.5 – Find all zeros

(a) $f(x) = -x^3 + x^2 + 2x$

(b) $f(x) = x^3 - x^2 + 9x - 9$

2.6 – Rational function applications

(a) Suppose that he insect population in millions in modeled by $(x) = \frac{10x+1}{0.2x+1}$, where $x \geq 0$ is in months. What happens to the insect population after a long time (in the long run)?

(b) A company that manufactures calculators has determined that the average cost for producing x calculators is $\bar{C} = \frac{15000+20x}{x}$ dollars. In the long run, what value does the average cost approach?

3.4 – Solve exponential and logarithmic equations. Give the exact answers.

(a) $3^{2x} - 3^{x} - 42 = 0$

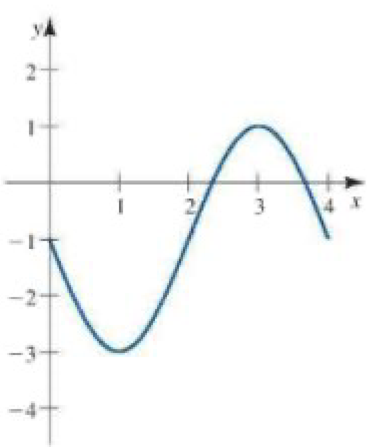
(b) $5^{x} = 3^{x-1}$

(c) $\log_2(x) + \log_2(x - 7) = 3$

(d) $\ln(x) - \ln(x - 2) = 1$

4.5 (a) – Find the amplitude, period and phase shift for the function: $y = -3\cos(2x + \pi)$

4.5 (b) – Find a sinusoidal model (function) for the given graph:



5.1 – Verify the trigonometric identities

(a) $\cos x \cot x + \sin x = \csc x$

(b) $\frac{\cos x + \sin x - \sin^3 x}{\sin x} = \cot x + \cos^2 x$

4.2, 5.2, 5.3 – Given $\sin \alpha = -\frac{3}{8}$, $\pi < \alpha < \frac{3\pi}{2}$ and $\cos \beta = \frac{3}{5}$, $0 < \beta < \frac{\pi}{2}$, find the following

(a) $\cos \alpha$ (b) $\sec \alpha$ (c) $\tan \alpha$ (d) $\cot \alpha$

(e) $\csc \alpha$ (f) $\cos(2\alpha)$ (g) $\sin(2\alpha)$ (h) $\tan(2\alpha)$

(i) $\cos\left(\frac{\alpha}{2}\right)$ (j) $\sin\left(\frac{\alpha}{2}\right)$ (k) $\sin(\alpha - \beta)$ (l) $\cos(\alpha + \beta)$

5.5 – Solve trigonometric equations on $[0,2\pi)$

(a) $\sin(2x) + \sqrt{2}\cos x = 0$

(b) $2\sin^2 x - 5\sin x + 2 = 0$

6.1 – Solve an application using Law of Sines

An aircraft is spotted by two observers who are 5000 meters apart. As the airplane passes over the line joining the observers, each observer takes a sighting of the angle of elevation of the airplane. The first observer sights the plane at $40°$ and the second observer sights the plane at $35°$.
How far away is the airplane from the first observer?

6.2 – Solve an application using Law of Cosines

A tourist stands 100 feet from the base of the Leaning Tower of Pisa. With the tower leaning away from the observer, the observer looking up at an angle of $52°$ finds that the distance from the top of the tower to where he is standing is 228 feet.
Find the angle the Leaning Tower makes with the ground.

Final Exam Review – Answers

1.2 – Find the domain

(a) $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$ (b) $\left(-\infty,\frac{2}{3}\right)\cup\left(\frac{2}{3},\infty\right)$

(c) $\left(-\frac{1}{2},\infty\right)$ (d) $(-\infty,4]$

1.3 – Evaluate the difference quotient

(a) $-2x+5-h$ (b) $-\frac{1}{3x(x+h)}$ (c) $6x+4+3h$

1.6 – Find the function $g(x)$ after applying the following transformations to x^2:

$$g(x) = -2\left(\frac{1}{4}(x+5)\right)^2 + 3$$

1.7 – Find compositions

(a) $(f\circ g)(x) = 4x^2 - 14x + 16$ and $(g\circ f)(x) = 2x^2 - 2x + 5$

(b) $(g\circ f)(x) = 2x$

1.8 – Find inverse functions

(a) $f^{-1}(x) = \frac{3x}{4-x}$ (b) $f^{-1}(x) = \sqrt[3]{x+10}$

2.2 – Quadratic function application

Maximum height is $\frac{539}{6} \approx 89.8$ feet, which occurs after $\frac{50}{9} \approx 5.56$ seconds; approximately 11.3 seconds after being thrown, the ball falls to the ground.

2.3-2.5 – Find all zeros

(a) $x = 2, 0, -1$ (b) $x = 1, 3i, -3i$

2.6 – Rational function applications

(a) The insect population approaches 50 million

(b) In the long run, the average cost approaches 20 dollars

3.4 – Solve exponential and logarithmic equations

(a) $x = \log_3 7 = \frac{\ln 7}{\ln 3} \quad (\approx 1.77)$

(b) $x = \frac{\ln 3}{\ln 3 - \ln 5} \quad (\approx -2.15)$

(c) $x = 8$

(d) $x = \frac{2e}{e-1} \quad (\approx 3.16)$

4.5 (a) – Find the amplitude, period and phase shift for the function: $y = -3\cos(2x + \pi)$

amplitude = 3, period $= \frac{2\pi}{2} = \pi$, phase shift $= -\frac{\pi}{2}$

4.5 (b) – Find a sinusoidal model (function) for the given graph:

- If modeled with reflected sine, there is no phase shift: $y = -2\sin\left(\frac{\pi}{2}t\right) - 1$
- If modeled with reflected cosine, the phase shift is 1 unit to the right: $y = -2\cos\left(\frac{\pi}{2}(t-1)\right) - 1$
- If modeled with sine, the phase shift is 2 units to the right: $y = 2\sin\left(\frac{\pi}{2}(t-2)\right) - 1$
- If modeled with cosine, the phase shift is 3 units to the right: $y = 2\cos\left(\frac{\pi}{2}(t-3)\right) - 1$

5.1 – Verify the trigonometric identities

(a)
$$\begin{aligned}
\cos x \cot x + \sin x &= \cos x \frac{\cos x}{\sin x} + \sin x \\
&= \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x} \\
&= \frac{\cos^2 x + \sin^2 x}{\sin x} \\
&= \frac{1}{\sin x} \\
&= \csc x
\end{aligned}$$

(b)
$$\begin{aligned}
\frac{\cos x + \sin x - \sin^3 x}{\sin x} &= \frac{\cos x}{\sin x} + \frac{\sin x}{\sin x} - \frac{\sin^3 x}{\sin x} \\
&= \cot x + \frac{\sin x - \sin^3 x}{\sin x} \\
&= \cot x + \frac{\sin x(1 - \sin^2 x)}{\sin x} \\
&= \cot x + 1 - \sin^2 x \\
&= \cot x + \cos^2 x
\end{aligned}$$

4.2, 5.2, 5.3 – Given $\sin\alpha = -\frac{3}{8}$, $\pi < \alpha < \frac{3\pi}{2}$ and $\cos\beta = \frac{3}{5}$, $0 < \beta < \frac{\pi}{2}$, find the following

(a) $\cos\alpha = -\frac{\sqrt{55}}{8}$ (b) $\sec\alpha = -\frac{8\sqrt{55}}{55}$ (c) $\tan\alpha = \frac{3\sqrt{55}}{55}$

(d) $\cot\alpha = \frac{\sqrt{55}}{3}$ (e) $\csc\alpha = -\frac{8}{3}$ (f) $\cos(2\alpha) = \frac{23}{32}$

(g) $\sin(2\alpha) = \frac{3\sqrt{55}}{32}$ (h) $\tan(2\alpha) = \frac{3\sqrt{55}}{23}$ (i) $\cos\left(\frac{\alpha}{2}\right) = -\frac{\sqrt{8-\sqrt{55}}}{4}$

(j) $\sin\left(\frac{\alpha}{2}\right) = \frac{\sqrt{8+\sqrt{55}}}{4}$ (k) $\sin(\alpha - \beta) = \frac{-9+4\sqrt{55}}{40}$ (l) $\cos(\alpha + \beta) = \frac{-3\sqrt{55}+12}{40}$

5.5 – Solve trigonometric equations on $[0,2\pi)$

(a) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{4}, \frac{7\pi}{4}$ (b) $x = \frac{\pi}{6}, \frac{5\pi}{6}$

6.1 – Solve an application using Law of Sines

2969.05 meters

6.2 – Solve an application using Law of Cosines

102.7°