MAT 171 – Exam 2 Review Problems

1. Write the exponential equations in logarithmic form:
   
   (a) \( a^{x+1} = 65 \)  
   
   (b) \( e^{3x} = 5 \)

2. Write the logarithmic equations in exponential form:
   
   (a) \( \log_6(4x) = 10 \)  
   
   (b) \( \ln B = A \)

3. Use properties of logs to write expressions as a sum or difference of logs with no exponents.
   
   (a) \( \log \left( \frac{x^5y^7}{z^3} \right) \)  
   
   (b) \( \ln \left( (x - 1)^{\frac{3}{2}} \sqrt[3]{\frac{(y+3)^4}{z^8}} \right) \)

4. Use properties of logs to write expressions as a single logarithm:
   
   (a) \( 3 \log A - 4 \log B + 5 \log C - 6 \log D \)  
   
   (b) \( \frac{3}{2} \ln(x + 3) - \ln(x) - \frac{1}{2} \ln(x + 3) \)
5. Find the accumulated value of an investment of $21,000 at an interest rate of 5.6% for 7 years
   (a) compounded monthly                      (b) compounded continuously

6. What initial investment at 4.25% interest compounded quarterly for 5 years will accumulate to $3,500? Round your answer to the nearest cent.

7. Solve the exponential equations
   (a) $2^{2x+17} = 8$                      (b) $(7)^{2x} + 2(7)^x - 15 = 0$          (c) $3^x = 7^{2-x}$
8. Solve the logarithmic equations

(a) \(2 \log_3(2x - 10) = 4\) 
(b) \(\ln x + \ln(2x + 1) = 0\)

(c) \(\log_2(x - 3) - \log_2(5x - 1) = 4\)

9. (a) Find the domain of the logarithmic function \(f(x) = \log(4x + 16)\), and find its inverse \(f^{-1}(x)\).

(b) Consider \(g(x) = 5^{x-2} + 1\). Find its inverse \(g^{-1}(x)\). What is the asymptote of the inverse?
10. The population of Merchantville was 20,000 in 1990 and 25,000 in 1995. Find an exponential function that models population growth and use it to estimate the population in the year 2008.

11. A sample of 500 grams of radioactive lead 210 decays to polonium 210 according to the function $A(t) = 500e^{-0.032t}$ where $t$ is in years. Find the half-life.

12. Find the length of the arc on a circle with radius 16 in intercepted by a central angle $\theta = 60^\circ$.

13. If the length of the arc on a circle of radius 10 cm is 20 cm, find the measure of the central angle in degrees.
14. Find a positive angle less than $360^\circ$ that is coterminal with the angle $-760^\circ$.

15. Find a positive angle less than $2\pi$ that is coterminal with the angle $\frac{17\pi}{3}$.

16. Given that $\cos t = -\frac{3}{5}$ and $t$ is in standard position with terminal side in quadrant III, find the exact values for $\sin t$, $\csc t$, $\sec t$, $\tan t$, $\cot t$.

17. A telephone pole is 55 feet tall. How long should a guy wire be if it is to be attached 15 feet from the top and is to make an angle of $35^\circ$ with the ground?

18. Find the reference angle for $-250^\circ$.

19. Evaluate exactly, rationalize if necessary.

(a) $\cos \left(\frac{5\pi}{6}\right)$

(b) $\cot(3\pi)$

(c) $\sec \left(\frac{5\pi}{4}\right)$

(d) $\tan \left(-\frac{\pi}{3}\right)$
Exam 2 Review – Answers

1. (a) \( \log_a(65) = x + 1 \)  \hspace{1cm} (b) \( \ln(5) = 3x \)

2. (a) \( 4x = 6^{10} \)  \hspace{1cm} (b) \( B = e^A \)

3. (a) \( 5 \log x + 7 \log y - 3 \log z \)  \hspace{1cm} (b) \( \frac{3}{2} \ln(x - 1) + 2 \ln(y + 3) - 4 \ln z \)

4. (a) \( \log \left( \frac{A^3 C^5}{B^4 D^6} \right) \)  \hspace{1cm} (b) \( \ln \left( \frac{x + 3}{x} \right) \)

5. (a) \$31,050.37  \hspace{1cm} (b) \$31,078.69

6. \$2,833.14

7. (a) \( x = -7 \)  \hspace{1cm} (b) \( x = \frac{\ln 3}{\ln 7} \approx 0.56458 \)  \hspace{1cm} (c) \( x = \frac{2 \ln 7}{\ln 3 + \ln 7} \approx 1.2783 \)

8. (a) \( x = \frac{19}{2} \)  \hspace{1cm} (b) \( x = \frac{1}{2} \)  \hspace{1cm} (c) No solution

9. (a) Domain: \((-4, \infty)\)  \hspace{1cm} Inverse: \( f^{-1}(x) = \frac{1}{4} 10^x - 4 \)

(b) Inverse: \( g^{-1}(x) = 2 + \log_5(x - 1) \)  \hspace{1cm} Asymptote: \( x = 1 \)

10. \( P(t) = 20,000e^{0.044629t} \)  \hspace{1cm} \( P(18) = 44,658 \)

11. 21.66 years

12. \( s = \frac{16\pi}{3} \) in. \( \approx 16.76 \) in.

13. \( \theta = 114.59^\circ \)

14. 320°

15. \( \frac{5\pi}{3} \)

16. \( \sin t = -\frac{4}{5}, \quad \csc t = -\frac{5}{4}, \quad \sec t = -\frac{5}{3}, \quad \tan t = \frac{4}{3}, \quad \cot t = \frac{3}{4} \)

17. 69.7 feet

18. 70°

19. (a) \( -\frac{\sqrt{3}}{2} \)  \hspace{1cm} (b) Undefined  \hspace{1cm} (c) \( -\sqrt{2} \)  \hspace{1cm} (d) \( -\sqrt{3} \)