MAT 171 – Exam 1 Review Problems

1. Use the given graph to find the following:

(a) domain
(b) range
(c) all x values that satisfy \( f(x) = -\frac{3}{2} \)

2. Find the domain of the following functions:

(a) \( f(x) = \frac{1-x}{x^2+x-6} \)
(b) \( f(x) = \sqrt{8-2x} \)

3. Given the function \( f(x) = \frac{x^3}{3} - 4x + 5 \), find the following:

(a) open intervals where \( f(x) \) is increasing, decreasing and constant.

(b) local maximum and minimum points.
4. Evaluate the difference quotient \( \frac{f(x+h)-f(x)}{h}, h \neq 0 \)

   (a) \( f(x) = 3x^2 - 5x + 2 \) \quad (b) \( g(x) = \frac{5}{x} \)

5. Evaluate the piece-wise function and sketch a graph.

\[
\begin{align*}
f(x) &= \begin{cases} 
\sqrt{4-x} & x < -3 \\
3x & x \geq -3 
\end{cases} \\
f(-2) &= \\
f(-5) &=
\end{align*}
\]

6. Determine algebraically or graphically if the following functions are even, odd or neither.

   (a) \( f(x) = 2x^2 - 7 \) \quad (b) \( g(x) = x^3 + 3x - 5 \) \quad (c) \( h(x) = \sqrt[3]{x} \)
7. (a) Find the function $g(x)$ after applying the following transformations to $x^2$:

reflect about the $x$-axis, shift left 5 units, shift up 3 units

(b) Use transformations on $x^2$ to graph $f(x)$ and describe the transformations in words:

\[ f(x) = 3(x - 1)^2 \]

8. Given $f(x) = 3x^2 + x - 1$ and $(x) = 3x - 2$, find the following:

(a) $(f - g)(x)$

(b) $\left(\frac{f}{g}\right)(x)$

(c) $(f \circ g)(x)$

(d) $(g \circ f)(2)$

9. Find inverse functions for the following one-to-one functions, and find the domain and range of each inverse:

(a) $f(x) = \frac{x}{2x - 3}$

(b) $f(x) = x^2 - 1$, $x \geq 0$
10. For the quadratic function \( f(x) = -2x^2 - 12x - 17 \), find:

(a) vertex  
(b) axis of symmetry  
(c) y-intercept  
(d) x-intercept(s)

11. A projectile is shot from a 350 foot cliff. The quadratic function \( s(t) = -15t^2 + 90t + 350 \) models the projectile’s height above ground, in feet, \( t \) seconds after it was shot. When does it reach its maximum height? What is the maximum height? Find the number of seconds it takes for the projectile to reach the ground. (Round to the nearest tenth)

12. Find zeros, list their multiplicities, and determine end-behavior of each polynomial function:

(a) \( f(x) = x^3 - 7x^2 + 11x \)  
(b) \( f(x) = 2x^3 - 3x^2 - 12x + 20 \)
13. Find all zeros of \( p(x) = x^3 + 10x^2 + 41x + 50 \)

14. Find a 3\(^{rd}\) degree polynomial function with zeros \(-\frac{2}{3}\) and \(1 - 4i\) and \(p(0) = 17\).

15. The following rational function models the population of a certain species of animal, in hundreds, where \(t\) is in days. What number does the population approach in the long run?

\[
P(x) = \frac{10x^3+2}{2x^3+1}
\]

16. Find all asymptotes and holes of the following functions:

(a) \( R(x) = \frac{3x+2}{x^2-1} \)  
(b) \( f(x) = \frac{4x^3}{x^2-7x} \)
Exam 1 Review – Answers

1. (a) $(-1,2]$ (b) $[-2,2]$ (c) $x = -0.7$ and $x = 0.7$ (answers will vary slightly)

2. (a) $(-\infty, -3) \cup (-3,2) \cup (2, \infty)$ (b) $(-\infty, 4]$ 

3. (a) increasing: $(-\infty, -2) \cup (2, \infty)$ decreasing: $(-2,2)$
   (b) maximum: $\left(-2, \frac{31}{3}\right)$ minimum: $\left(2, -\frac{1}{3}\right)$ 

4. (a) $6x + 3h - 5$ (b) $\frac{5}{x^2 + xh}$ 

5. $(-2) = -6, \ f(-5) = 3$

6. (a) even (b) neither (c) odd 

7. (a) $g(x) = -(x + 5)^2 + 3$
   (b) vertical stretch by a factor of 3 and a horizontal shift right by 1 unit 

Dashed red: $y = x^2$

Solid black: $f(x) = 3(x - 1)^2$
8. (a) \((f - g)(x) = 3x^2 - 2x + 1\)  
     (b) \(\left( \frac{f}{g} \right)(x) = \frac{3x^2 + x - 1}{3x - 2}\) for \(x \neq \frac{2}{3}\)  
     (c) \((f \circ g)(x) = 27x^2 - 33x + 9\)  
     (d) \((g \circ f)(2) = 37\)

9. (a) \(f^{-1}(x) = \frac{3x}{2x - 1}\)  
     domain: \((-\infty, \frac{1}{2}) \cup \left(\frac{1}{2}, \infty\right)\)  
     range: \((-\infty, \frac{3}{2}) \cup \left(\frac{3}{2}, \infty\right)\)  
     (b) \(f^{-1}(x) = \sqrt{x + 1}\)  
     domain: \([-1, \infty)\)  
     range: \([0, \infty)\)

10. (a) \((-3, 1)\)  
     (b) \(x = -3\)  
     (c) \((0, -17)\)  
     (d) \(\left(\frac{-6 + \sqrt{2}}{2}, 0\right), \left(\frac{-6 - \sqrt{2}}{2}, 0\right)\)

11. 485 feet, 3 seconds, 8.7 seconds

12. (a) \(x = 0, \frac{7}{2} \pm \frac{1}{2}\sqrt{5}\), all multiplicity 1; graph rises to the right and falls to the left  
     (b) \(x = -\frac{5}{2}\) with \(m = 1\) and \(x = 2\) with \(m = 2\); graph rises to the right and falls to the left

13. \(x = -2, -4 + 3i, -4 - 3i\)

14. \(p(x) = \frac{3}{2}x^3 - 2x^2 + \frac{47}{2}x + 17\)

15. 5 hundred

16. (a) horizontal asymptote \(y = 0\) and vertical asymptotes \(x = 1, x = -1\)  
     (b) no horizontal asymptote, vertical asymptote \(x = 7\) and hole at \(x = 0\)