Answers: MAT 210 Exam 2 Review Questions

Chain Rule (section 11.4)

1.
   a) $2(3x^2 + 2x - 8)^4(6x + 2)$
   b) $\frac{1}{2}(x^3 - 50x)^{-\frac{1}{2}} \cdot (3x^2 - 50) = \frac{3x^2 - 50}{2\sqrt{x^3 - 50x}}$
   c) $\frac{4}{3}(6x + 1)^{1/3} \cdot 6 = 8(6x + 1)^{\frac{1}{3}} = 8\sqrt[3]{6x + 1}$
   d) $25(-4)(x^2 + x + 2)^{-5} \cdot (2x + 1) = -100(2x + 1)(x^2 + x + 2)^{-5} = -\frac{100(2x + 1)}{(x^2 + x + 2)^5}$

Derivative of Logarithmic and Exponential Functions (section 11.5)

1.
   a) $9 \cdot \frac{1}{2x} \cdot 2 = \frac{9}{x}$
   b) $\frac{15x^2 + 2x}{5x^3 + x^2 + 4}$
   c) $\frac{3x^2 - 8}{x^3 - 8x}$
   d) $1 - \left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) = -\ln x$
   e) $4e^{x^5 - 3x} \cdot (5x^4 - 3)$
   f) $(2x - 2)e^{2x + 3} + (x^2 - 2x)e^{2x + 3} \cdot 2 = (2x^2 - 2x - 2)e^{2x + 3}$
   g) $(-5x^{-2})e^{5/x}$
   h) $-16e^{-2x}$
   i) $5(9x^2 + 2)e^{3x^3 + 2x}$

Implicit Differentiation (section 11.6)

1.
   a) $\frac{dy}{dx} = \frac{3x^2}{3y^2 - 1}$
   b) $\frac{dy}{dx} = \frac{15 - 12xy}{6x^2 - 2y}$
   c) $\frac{dy}{dx} = \frac{e^x - e^y}{xe^y}$
Maxima and Minima (section 12.1)

1. 
   (a) −3, 0, 3
   (b) −1, 1
   (c) −3, 3
   (d) −1, 0, 1
   (e) (−3, −1) and (0, 1)
   (f) (−1, 0) and (1, 3)

2. 
   (a) −2, 0, 2
   (b) −2; 2

3. Critical point x = 2; f has a relative minimum at x = 2, which is equal to f(2) = 2 − 2 ln 2.

4. 
   (a) Critical points: x = −1, 1.
   (b) Abs Max = f(−1) = f(2) = 28; Abs Min = f(−3) = −132.

5. 
   (a) Critical point: x = 2
   (b) Abs Max = g(5) = 3^{2/3}; Abs Min = g(2) = 0.

6. 
   (a) Critical points: x = 1, 3.
   (b) Increasing on (−∞, 1) ∪ (3, ∞): decreasing on (1, 3).
   (c) Abs Max = k(5) = 60; Abs Min = k(−1) = −84; Rel Max are k(1) = −4 and k(5) = 60;
      Rel Min are k(3) = −20 and k(−1) = −84.

7. 
   (a) Critical points: x = 0.
   (b) Increasing on (0, 3); decreasing on (−2, 0).
   (c) Abs Max = h(3) = e^3 − 3; Abs Min = h(0) = 1; Rel Max are h(−2) = e^{−2} + 2 and h(3) = e^3 − 3;
      Rel Min is h(0) = 1.

8. T, T, F, T.

9. 
   (a) No relative extrema
   (b) Relative min
   (c) (4, ∞)
   (d) (−∞, 4)
Optimization: Applications to Maximum and Minimum (Section 12.2)

1. (a) \( P(q) = 68q - 0.1q^2 - (23 + 20q) = -0.1q^2 + 48q - 23 \)
   (b) Set \( P' = -0.2q + 48 = 0 \), and solve for \( q: q = 240 \)
   (c) \( P(240) = -0.1 \cdot 240^2 + 48 \cdot 240 - 23 = 5737 \) dollars.

2. \( \bar{C}(x) = 0.02x + 2 + 4000x^{-1} \). Set \( \bar{C}'(x) = 0.02 - 4000x^{-2} = 0 \) and solve for \( x: x = 447 \) units.

3. Maximize area \( A = xy \) subject to cost \( 3x + 10y = 120 \), where \( x \) is the length of the north and south sides, and \( y \) is the length of east and west sides.
   Dimension is \( x = 20 \) ft, \( y = 6 \) ft. Max area = 120 square feet.

4. Minimize the cost \( C = 4x + 8y \) subject to area \( xy = 162 \), where \( x \) is the length of the north and south sides, and \( y \) is the length of east and west sides.
   Dimension is \( x = 18 \) ft, \( y = 9 \) ft. Minimum cost = 144 dollars.

5. \( R(p) = p \cdot q = p(-10p + 4220) = -10p^2 + 4220p \). Set \( R'(p) = -20p + 4220 = 0 \) and solve for \( p: p = $211 \).
   Maximum revenue is \( R(211) = 211 \cdot 2110 = $445210 \).

Higher-order Derivatives, Acceleration and Concavity (Section 12.3)

1. (a) \( y'' = 8e^{2x-5} \)
   (b) \( y'' = 14x^{-3} + 5x^{-2} \)

2. (a) 0; (b) negative, positive; (c) 0; (d) down, up.

3. (a) down, up, down
   (b) \((2, 3), (-\infty, 2) \cup (3, \infty)\);
   (c) \( x = 2, 3 \)
   (d) Relative max at \( x = 3.5 \), no relative min.
   (e) Abs max at \( x = 3.5 \), no abs min.

4. \( a(t) = s''(t) = -\frac{1}{4}t^{-3} + 8 \)

5. 8 ft/sec²

Related Rates (Section 12.5)

1. (a) 2827 cm²/sec; (b) 479 cm²/sec
2. 0.03 cm/sec
3. When \( x = 4000 \), \( \frac{d\bar{C}}{dt} = \frac{d\bar{C}}{dx} \cdot \frac{dx}{dt} = $ - 2.84 \) per player per week. So, the average cost is decreasing at a rate of 2.84 dollars per player per week.
Elasticity (Section 12.6) (Optional, this lesson will not be covered on the exam)

1. down, 3.2%, decrease.

2. down, 0.65%, increase.

3.

(a) \( E(p) = -(-18) \cdot \frac{p}{1080-18p} = \frac{18p}{1080-18p} \), so \( E(20) = \frac{18 \cdot 20}{1080-18 \cdot 20} = 0.5 \).

This means the demand will drop by 0.5% for 1% increase from current price $20.

(b) 0.5 < 1, it is inelastic. The price should be raised to increase revenue.

(c) Solve for the price when \( E(p) = 1 \). Solving \( \frac{18p}{1080-18p} = 1 \) gives \( p = $30 \).

(d) \( R = pq = 30(1080 - 18 \cdot 30) = 16200 \) dollars.

4.

(a) \( E(p) = -(-4p + 33) \cdot \frac{p}{-2p^2+33p} = \frac{4p-33}{-2p+33} \)

(b) \( E(15) = \frac{4(15)-33}{-2(15)+33} = \frac{27}{3} = 9 > 1 \). It is elastic. The demand will drop by 9% if the price increases by 1%.

The price should be lowered from $15 to increase revenue.