Answers: MAT 210 Exam 2 Review Questions

Chain Rule (section 11.4)

1.
   a) $2(3x^2+2x-8)^4(6x+2)$
   b) $\frac{1}{2}(x^3-50x)^{-\frac{1}{2}} \cdot (3x^2-50) = \frac{3x^2-50}{2\sqrt{x^3-50x}}$
   c) $\frac{4}{3}(6x+1)^{1/3} \cdot 6 = 8(6x+1)^{\frac{1}{3}} = 8\sqrt[3]{6x+1}$
   d) $25(-4)(x^2+x+2)^{-5} \cdot (2x+1) = -100(2x+1)(x^2+x+2)^{-5} = -\frac{100(2x+1)}{(x^2+x+2)^5}$

Derivative of Logarithmic and Exponential Functions (section 11.5)

1.
   (a) $9 \cdot \frac{1}{2x} \cdot 2 = \frac{9}{x}$
   (b) $\frac{15x^2+2x}{5x^3+x^2+4}$
   (c) $\frac{3x^2-8}{x^3-8x}$
   (d) $1-\left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) = -\ln x$
   (e) $4e^{x^5-3x} \cdot (5x^4-3)$
   (f) $(2x-2)e^{2x+3} + (x^2-2x)e^{2x+3} \cdot 2 = (2x^2-2x-2)e^{2x+3}$
   (g) $(-5x^{-2})e^{5/x}$
   (h) $-16e^{-2x}$
   (i) $5(9x^2+2)e^{3x^3+2x}$

Implicit Differentiation (section 11.6)

1.
   (a) $\frac{dy}{dx} = \frac{3x^2}{3y^2-1}$
   (b) $\frac{dy}{dx} = \frac{15-12xy}{6x^2-2y}$
   (c) $\frac{dy}{dx} = \frac{e^x-e^y}{xe^y}$

Maxima and Minima (section 12.1)

1.
   (a) $-3, 0, 3$
   (b) $-1, 1$
   (c) $-3, 3$
   (d) $-1, 0, 1$
   (e) $(-3, -1)$ and $(0, 1)$
   (f) $(-1, 0)$ and $(1, 3)$

2.
   (a) $-2, 0, 2$
   (b) $-2; 2$

3. Critical point $x = 2$; f has a relative minimum at $x = 2$, which is equal to $f(2) = 2 - 2\ln 2$.

4.
   (a) Critical points: $x = -1, 1$.
   (b) Abs Max = $f(-1) = f(2) = 28$; Abs Min= $f(-3) = -132$.

5.

   (a) Critical point: $x = 2$
   (b) Abs Max = $g(5) = 3^{2/3}$; Abs Min = $g(2) = 0$.

6.
   (a) Critical points: $x = 1, 3$.
   (b) Increasing on $(-\infty, 1) \cup (3, \infty)$; decreasing on $(1, 3)$.
   (c) Abs Max = $k(5) = 60$; Abs Min = $k(-1) = -84$; Rel Max are $k(1) = -4$ and $k(5) = 60$; Rel Min are $k(3) = -20$ and $k(-1) = -84$.

7.
   (a) Critical points: $x = 0$.
   (b) Increasing on $(0, 3)$; decreasing on $(-2, 0)$.
   (c) Abs Max = $h(3) = e^3 - 3$; Abs Min = $h(0) = 1$; Rel Max are $h(-2) = e^{-2} + 2$ and $h(3) = e^3 - 3$; Rel Min is $h(0) = 1$.

8. **T, T, F, T.**
9.
   (a) No relative extrema
   (b) Relative min
   (c) $(4, \infty)$
   (d) $(-\infty, 4)$

Optimization: Applications to Maximum and Minimum (Section 12.2)

1.
   (a) $P(q) = 68q - 0.1q^2 - (23 + 20q) = -0.1q^2 + 48q - 23$
   (b) Set $P' = -0.2q + 48 = 0$, and solve for q: $q = 240$
   (c) $P(240) = -0.1 \cdot 240^2 + 48 \cdot 240 - 23 = 5737$ dollars.

2. $\bar{C}(x) = 0.02x + 2 + 4000x^{-1}$. Set $\bar{C}'(x) = 0.02 - 4000x^{-2} = 0$ and solve for x: $x = 447$ units.

3. Maximize area $A = xy$ subject to cost $3x + 10y = 120$, where x is the length of the north and south sides, and y is the length of east and west sides.

   Dimension is $x = 20\ ft, y = 6\ ft$. Max area = 120 square feet.

4. Minimize the cost $C = 4x + 8y$ subject to area $xy = 162$, where x is the length of the north and south sides, and y is the length of east and west sides.

   Dimension is $x = 18\ ft, y = 9\ ft$. Minimum cost = 144 dollars.

5. $R(p) = p \cdot q = p(-10p + 4220) = -10p^2 + 4220p$. Set $R'(p) = -20p + 4220 = 0$ and solve for p: $p = \$\ 211$. Maximum revenue is $R(211) = 211 \cdot 2110 = \$445210$.

Higher-order Derivatives, Acceleration and Concavity (Section 12.3)

1.
   (a) $y'' = 8e^{2x-5}$
   (b) $y'' = 14x^{-3} + 5x^{-2}$

2. (a) 0; (b) negative, positive; (c) 0; (d) down, up.
3.
   (a) down, up, down
   (b) $(2,3)$, $(-\infty, 2) \cup (3, \infty)$;
   (c) $x = 2, 3$
   (d) Relative max at $x = 3.5$, no relative min.
   (e) Abs max at $x = 3.5$, no abs min.

4. $a(t) = s''(t) = -\frac{1}{4}t^{-\frac{3}{2}} + 8$
5. 8 $\mathrm{ft/sec^2}$

Related Rates (Section 12.5)

1. (a) 2827 $\mathrm{cm^2/sec}$; (b) 479 $\mathrm{cm^2/sec}$
2. 0.03 cm/sec
3. When $x = 4000$, $\frac{d\bar{C}}{dt} = \frac{d\bar{C}}{dx} \cdot \frac{dx}{dt} = \$ - 2.84$ per player per week. So, the average cost is decreasing at a rate of 2.84 dollars per player per week.

Elasticity (Section 12.6) (Optional, this lesson will not be covered on the exam)

1. down, 3.2%, decrease.

2. down, 0.65%, increase.

3.

(a) $E(p) = -(-18) \cdot \frac{p}{1080-18p} = \frac{18p}{1080-18p}$, so $E(20) = \frac{18 \cdot 20}{1080-18 \cdot 20} = 0.5$.
This means the demand will drop by 0.5% for 1% increase from current price \$20.

(b) $0.5 < 1$, it is inelastic. The price should be raised to increase revenue.

(c) Solve for the price when $E(p) = 1$. Solving $\frac{18p}{1080-18p} = 1$ gives $p = \$30$.

(d) $R = pq = 30(1080 - 18 \cdot 30) = 16200$ dollars.

4.

(a) $E(p) = -(-4p + 33) \cdot \frac{p}{-2p^2+33p} = \frac{4p-33}{-2p+33}$

(b) $E(15) = \frac{4(15)-33}{-2(15)+33} = \frac{27}{3} = 9 > 1$. It is elastic. The demand will drop by 9% if the price increases by 1%.
The price should be lowered from \$15 to increase revenue.