

M E M O R A N D U M

DATE: 05/22/2023

TO: Faculty and Students

FROM: Professor(s) Marilyn Carlson and Naneh Apkarian
Chair/Co-Chairs of Kayla Lock
Defense for the PhD in Mathematics Education
Committee Members April Strom
Cameron Byerley
Patrick Thompson

DEFENSE ANNOUNCEMENT

Candidate: Kayla Lock

Defense Date: Thursday, June 22, 2023

Defense Time: 10:00 AM

Virtual Meeting Link: <https://asu.zoom.us/j/87480662958> Or may attend live: WCLR 206

Title: Investigating the Role of Relative Size Reasoning in Understanding Precalculus Ideas

Please share this information with colleagues and other students, especially those studying in similar fields. Faculty and students are encouraged to attend. The defending candidate will give a 40-minute talk, after which the committee members will ask questions. There may be time for questions from those in attendance. However, guests are primarily invited to attend as observers and will be excused when the committee begins its deliberations or if the committee wishes to question the candidate privately.

ABSTRACT
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The ideas of measurement and measurement comparisons (e.g., fractions, ratios, quotients) are introduced to students in elementary school. However, studies report that students of all ages have difficulty comparing two quantities in terms of their relative size. Students often understand fractions such as $\frac{3}{7}$ as part-whole relationships or “three out of seven.” These limited conceptions have been documented to have implications for understanding the quotient as a measure of relative size and when learning other foundational ideas in mathematics (e.g., rate of change). Many scholars have identified students’ ability to conceptualize the relative size of two quantities values as important for learning specific ideas such as constant rate of change, exponential growth, and derivative. However, few researchers have focused on students’ ways of thinking about multiplicatively comparing two quantities’ values as they vary together across select topics in precalculus. Relative size reasoning is a way of thinking one has developed when conceptualizing the comparison of two quantities’ values multiplicatively, as their values vary in tandem.

This document reviews literature related to relative size reasoning and presents a conceptual analysis that leverages this research in describing what I mean by a relative size comparison and what it means to engage in relative size reasoning. I further illustrate the role of relative size reasoning in understanding rate of change, multiplicative growth, rational functions, and what a graph’s concavity conveys about how two quantities’ values vary together.

This study reports on three beginning calculus students’ ways of thinking as they completed tasks designed to elicit students’ relative size reasoning. The data revealed 4 ways of conceptualizing the idea of quotient and highlights the affordances of conceptualizing a quotient as a measure of the relative size of two quantities’ values. The study also reports data from

investigating the validity of a collection of multiple-choice items designed to assess students' relative size reasoning (RSR) abilities. Analysis of this data provided insights for refining the questions and answer choices for these assessment items.

This study has extended previous research by (a) documenting the ways of thinking entailed in engaging in relative size reasoning; (b) documenting four conceptions of quotient and highlighting the affordances of understanding a quotient as a measure of the relative size of two quantities' values as they vary in tandem; (c) characterizing the role of relative size reasoning when learning select ideas in precalculus, and (d) producing insights for refining items for a relative size reasoning (RSR) assessment instrument.