MAT 265 Exam Three Review   Sections: 4.1-4.5, 4.7, 5.1, 5.2

Correct Answer is highlighted red - 35 questions

Section 4.1

1. For $x > 0$, find the x-coordinate of the absolute minimum for the function $f(x) = 10x\ln(x) - 11x$.

2. The function $f(x) = (7x+5)e^{-6x}$ has one critical number. Find it.

3. Consider the function $f(x) = x^3 - 6x^2 - 63x + 8$ on $[-4, 8]$. Use the Closed Interval Method to find the absolute maximum and absolute minimum and the location of each. Show your work

4. For $x > 0$, find the x-coordinate to 4 decimal places of the absolute maximum for the function $f(x) = \frac{2+5\ln(x)}{x}$. Justify that your answer is an absolute minimum using calculus.

5. Find, to 4 decimal places, the critical numbers of the function $f(x) = \frac{5x}{9x^2+7}$.

Section 4.2

6. Verify with Rolle's Theorem, the function $f(x) = x^2 - 4x + 8$ on the interval $[0, 4]$ satisfies Rolle's Theorem. Find the value of c that is guaranteed by Rolle's Theorem.

7. Consider the function $f(x) = 2x^3 + 4x^2 + x - 4$ on the interval $[2, 5]$. Find the value(s) of c that satisfies the conclusion of the Mean Value Theorem to four decimal places.

8. Consider the function $f(x) = 4 - 6x^2$ on the interval $[-2,5]$. Find the value(s) of c that satisfies the conclusion of the Mean Value Theorem to four decimal places.

9. At 2:00pm a car's speedometer reads 50 mph, and at 2:10 pm it reads 80 mph. Use the Mean Value Theorem to find an acceleration the car *must* achieve.

10. Consider the function $f(x) = \frac{1}{x}$ on the interval $[1,12]$. Find the value(s) of c that satisfies the conclusion of the Mean Value Theorem to four decimal places.

Section 4.3

11. For the function $f(x) = (9 - 2x)e^{3x}$, list the x-values of the inflection point.

12. Suppose that $f(x) = 2x^5 - 5x^4$. Use interval notation to indicate where $f(x)$ is increasing and where it is decreasing.

13. Suppose that $f(x) = \frac{e^x}{4 + e^x}$. Use interval notation to indicate where $f(x)$ is concave up and concave down. Justify your answer with the second derivative.

Section 4.4

14. Suppose that $f(x) = 5x - 2\ln(x)$; $x > 0$. Use interval notation to state where the function is concave up and concave down. Justify your answer with the second derivative.

15. Given the function: $f(x) = xe^{2x}$
Find the intervals where the function is concave up and those for which it is concave down.

16. Suppose that $f(x) = (2 - x)e^x$. Use interval notation to indicate where $f(x)$ is increasing and where it is decreasing.

Section 4.5

17. A fence is to be built to enclose a rectangular area of 320 square feet. The fence along three sides is to be made of material that costs 3 dollars per foot and the fourth side costs 14 dollars per foot. Find the width ($W \le L$) in feet of the enclosure that is most economical to construct to four decimal places.

18. A box is to be made out of a 10 cm by 16 cm piece of cardboard. Squares of side length x cm will be cut out of each corner, and then the ends and sides will be folded up to form a box with an open top. Find the maximum volume of the box.

19. A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 8 - x^2$. What are the dimensions of such a rectangle with the greatest possible area?

20. An open rectangular box is constructed such that the volume is 16 cubic ft. Find the minimum cost of constructing the box to the nearest penny. The length of the base is twice the width. The materials to construct the base cost $14 per square ft and the sides cost $6 per square ft.

21. Find two positive numbers whose product is 64 and whose sum is a minimum.

22. Find two numbers whose difference is 46 and whose product is minimized.

Section 4.7

23. Given $f'(x) = 3\cos(x) - 9\sin(x)$ and $f(0) = -2$. Find $f(x)$.

24. Find the particular antiderivative that satisfies the following conditions:

$$f'''(x) = e^x; \quad f''(0) = 2; \quad f'(0) = 9$$

25. Find the general antiderivative for $f(x) = \frac{100}{1+x^2}$.

26. Find the particular antiderivative that satisfies the following conditions:

$$f''(x) = 8x + 2\sin x \;; \quad f(0) = 2; \quad f'(0) = 3$$

27. A particle moves on a straight line and has acceleration $a(t) = 36t + 10$. Its position at time $t = 0$ is $s(0) = 8$ and its velocity at time $t = 0$ is $v(0) = 9$. What is its position at time $t = 6$?

Section 5.1

28. Estimate the area under the graph of $f(x) = 4x^3 + 5$ from $x = -1$ to $x = 5$ using 12 rectangles by finding a left-hand approximation and a right hand approximation.

29. What is the definite integral for this limit:

$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{8}{n} \sqrt{1 + \left(\frac{8i}{n}\right)^2}$$

30. Estimate the area under the graph of $f(x) = \tan(x^2)$ from $x = 0$ to $x = 1$ using 5 approximating rectangles and right endpoints to 2 decimal places.

31. Speedometer readings for a motorcycle at 12-seconds intervals are given in the table.

t(s)	0	12	24	36	48	60
v(ft/sec)	20	24	22	26	24	21

Estimate the distance traveled by the motorcycle during this time period using the velocities at the beginning of the time intervals.

Section 5.2

32. Evaluate the integral by interpreting it in terms of areas to two decimal places:

$$\int_{0}^{10} (|6x - 1|)\,dx$$

33. Evaluate the integral by interpreting it in terms of areas:

$$\int_{-4}^{4} (3 - |x|)\,dx$$

34. Evaluate the integral below by interpreting it in terms of areas.

$$\int_{-15}^{15} \sqrt{225 - x^2}\,dx$$

35. Find a and b if $\int_0^{10} f(x)dx - \int_0^{7} f(x)dx = \int_a^{b} f(x)dx$.

Answers

1. $e^{1/10}$
2. $-\frac{23}{42}$
3. ABS MIN -384 occurs at $x = 7$;   AB MAX 116 occurs at $x = -3$
4. 1.8221
5. $-0.8819, 0.8819$
6. $f(x)$ is continuous on $[0,4]$, differentiable on $(0,4)$, and $f(0) = f(4) = 8$.
   So there is a c with $f'(c) = 0$. $c = 2$
7. 3.5890
8. 1.5
9. 180mi/h^2
10. 3.4641
11. $\frac{23}{6}$
12. Increasing on $(-\infty, 0) \cup (2,\ \infty)$;   Decreasing on (0,2)
13. CU $(-\infty,\ \ln(4))$;   CD $(\ln(4),\ \infty)$
14. CU $(0, \infty)$ CD Nowhere
15. CU $(-1, \infty)$;   CD $(-\infty, -1)$
16. Increasing on $(-\infty, 1)$;   Decreasing on $(1,\ \infty)$
17. 10.6274
18. 144
19. $4\sqrt{\frac{2}{3}}, \frac{16}{3}$
20. $\$250.27$
21. 8, 8
22. $-23, 23$
23. $3\sin(x) + 9\cos(x) - 11$
24. $e^x + 0.5x^2 + 8x + C$
25. $100\arctan(x) + C$
26. $\frac{4}{3}x^3 - 2\sin x + 5x + 2 + C$
27. 1538
28. $L_{12} = 534, R_{12} = 786$

29. $\int_0^8 \sqrt{1+x^2}\,dx$

30. 0.58

31. 1392

32. 290.17

33. 8

34. $\frac{225\pi}{2}$

35. $a = 7;\ \ b = 10$