School of Mathematical and Statistical Sciences
Arizona State University

M E M O R A N D U M

DATE: 03/29/2023

TO: Faculty and Students

FROM: Professor(s) Florian Sprung Nancy Childress
Chair/Co-Chairs of Alexander Reamy
Defense for the PhD in Mathematics
Committee Members
Jonathan Montaño
Julien Paupert
Steven Kaliszewski

DEFENSE ANNOUNCEMENT

Candidate: Alexander Reamy

Defense Date: Friday, April 14, 2023

Defense Time: 2:00 PM

Virtual Meeting Link: https://asu.zoom.us/j/86172444438

Title: Generalizations of the Signed Selmer Groups for Cyclotomic Extensions

Please share this information with colleagues and other students, especially those studying in similar fields. Faculty and students are encouraged to attend. The defending candidate will give a 40 minute talk, after which the committee members will ask questions. There may be time for questions from those in attendance. But, guests are primarily invited to attend as observers and will be excused when the committee begins its deliberations or if the committee wishes to question the candidate privately.

ABSTRACT
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ABSTRACT

Let E/K be an elliptic curve and p a rational prime. Also let K_∞/K be the cyclotomic $\mathbb{Z}_p$-extension of K and $\Lambda(\Gamma)$ the Iwasawa module of this infinite extension. A famous conjecture by Mazur states that the p-primary component of the Selmer group of E is $\Lambda(\Gamma)$-cotorsion when E has good reduction at all primes of K lying over p. This conjecture was proven in the case $K = \mathbb{Q}$ by Kato, but is known to be false when E has supersingular reduction type. To salvage this result, Kobayashi introduced the signed Selmer groups, which impose stronger local conditions than their classical counterparts.

Part of the construction of the signed Selmer groups involves defining a canonical system of points that generates the $\mathbb{Z}_p$-modules $F(m_{n+1})/F(m_n)$, where the m_n's are the maximal ideals of the layers of the cyclotomic $\mathbb{Z}_p$- extension of $\mathbb{Q}_p$ and $F(X, Y)$ is a formal group law over $\mathbb{Z}_p$ that is strictly isomorphic to the formal group law associated with $E/\mathbb{Q}_p$. This presentation offers an alternate construction of the canonical system of points that appeals to the Functional Equation Lemma instead of Honda's theory of commutative formal groups, and explores a possible way of generalizing this method to elliptic curves defined over finite extensions $K/\mathbb{Q}_p$ by passing from formal group laws to formal modules.