

LIST OF CONCEPTS AND SKILLS FOR TEST 1

The test covers sections 1.1-1.3, 2.1-2.5, 2.7, 3.1, 3.2

Chapter 1

- Know how to sketch (BY HAND) a direction field for a first order ODE, and how to sketch integral curves.
- Know how to find Equilibrium (constant) Solutions of ODEs of the form $y' = f(y)$ and how to classify them as stable, unstable or semi-stable.
- Given the slope field for a differential equation, be able to read the long term behavior of the solutions for different initial conditions.
- Know how to verify by substitution that a given function is a solution of a given ODE.
- Given the general solution to an ODE, know how to use initial conditions to find the value of the constant C .
- Know how to classify differential equations by their order and linearity.
- Know how to derive differential equations that model simple applied problems.

Chapter 2

- Know how to use the method of integrating factor to integrate linear first order equations of the form $y' + p(t)y = g(t)$. The integrating factor is given by

$$\mu(t) = e^{\int p(t)dt}.$$

- Know how to solve separable equations. Whenever possible write the solution in explicit form and be able to determine the interval in which the solution is defined.
- Know the Theorems for existence and uniqueness of solutions to first-order initial value problems (Theorem 2.4.1 and Theorem 2.4.2) and how to apply them.
- Be able to set up and solve applied problems such as “mixture” problems, 1-D motion with air resistance and terminal velocity, and problems involving Newton's Law of Cooling.
- Know the formula for Euler's method and how to use it to derive recursive approximations for a given IVP.

Chapter 3 (3.1)

- A second order linear differential equation in standard form has the form:

$$y'' + p(t)y' + q(t)y = g(t).$$

- If $g(t) = 0$, it is called homogeneous; otherwise nonhomogeneous.

Linear Homogeneous DEs with constant coefficients. Given the homogeneous ODE

$$ay'' + by' + cy = 0$$

with a , b and c constant:

- The characteristic equation is $ar^2 + br + c = 0$.
- If the roots of the characteristic equation are real and distinct, r_1 and r_2 , then the general solution of the homogeneous ODE is $y = c_1e^{r_1t} + c_2e^{r_2t}$.
- Know how to determine the exact solution (solve for c_1 and c_2) of the above when given two initial conditions.

Chapter 3 (3.2) (If on Exam 1)

The Wronskian:

- Know Theorem 3.2.1: Existence and Uniqueness. Consider the Initial Value Problem

$$y'' + p(t)y' + q(t)y = g(t), \quad y(t_0) = y_0, \quad y'(t_0) = y'_0$$

where $p(t)$, $q(t)$ and $g(t)$ are continuous on an open interval I that contains t_0 . Then there is exactly one solution of this problem, and the solution exists throughout the interval I .

- Know Theorem 3.2.2: **Principle of Superposition.** If y_1 and y_2 are solutions of the homogeneous differential equation $y'' + p(t)y' + q(t)y = 0$, so is $c_1y_1 + c_2y_2$ for any constants c_1 and c_2 .
- Know how to find the Wronskian of two functions, y_1 and y_2 .
- Know Theorem 3.2.4: **Wronskian of Solutions:** Given two solutions y_1 and y_2 of a homogeneous ODE, always check whether the Wronskian of the two solutions is not everywhere zero. If this is case the two solutions are linearly independent and we say they form a Fundamental Set of Solutions for the ODE. The general solution is given by $c_1y_1 + c_2y_2$ with c_1 and c_2 arbitrary constants.