MAT 210 Exam 2 Review Questions

Chain Rule (section 11.4)

1. Find the derivative y' of each function.

(a) $y = 0.4(3x^2 + 2x - 8)^5$

(b) $y = \sqrt{x^3 - 50x}$

(c) $y = (6x + 1)^{4/3}$

(d) $y = \frac{25}{(x^2+x+2)^4}$

Derivative of Logarithmic and Exponential Functions (section 11.5)

1. Find the derivative y' of each function.

(a) $y = 9\ln(2x)$

(b) $y = \ln(5x^3 + x^2 + 4)$

(c) $y = \ln|x^3 - 8x|$

(d) $y = x - x\ln x$

(e) $y = 4e^{x^5 - 3x}$

(f) $y = (x^2 - 2x)e^{2x+3}$

(g) $y = e^{5/x}$

(h) $y = 8e^{-2x}$

(i) $y = 5e^{3x^3+2x}$

Implicit Differentiation (section 11.6)

1. Find the derivative $\frac{dy}{dx}$.

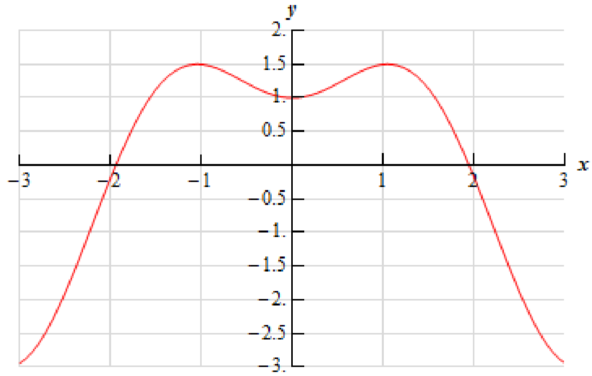
(a) $x^3 - y^3 + y = 3$

(b) $6x^2y - 15x = y^2$

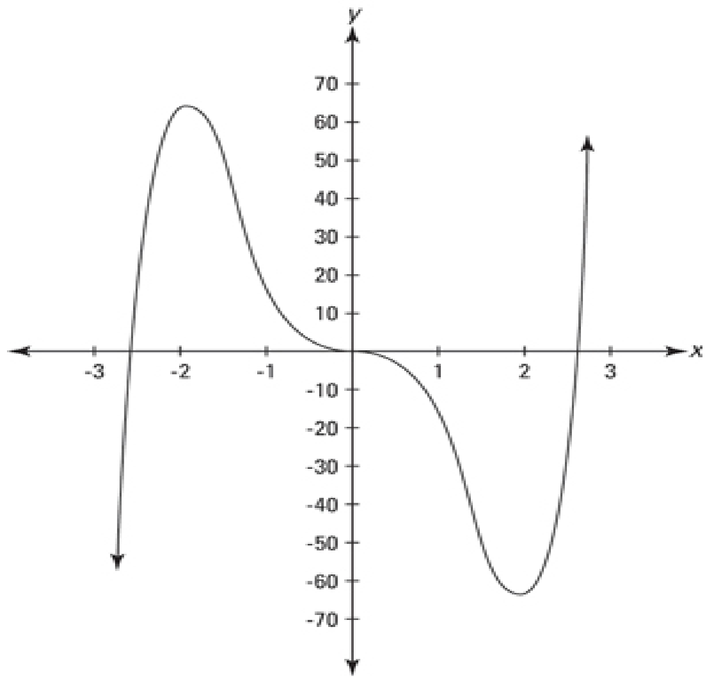
(c) $xe^y - e^x = 0$

Maxima and Minima (section 12.1)

1. The function of a function f on $[-3, 3]$ is given below.
   (a) f has a relative minimum at $x =$
   (b) f has an absolute maximum at $x =$
   (c) f has an absolute minimum at $x =$
   (d) f' is zero at $x =$
   (e) f' is positive on interval(s):
   (f) f' is negative on interval(s):



2. The graph of a function f is given below.
   (a) f' is zero at $x =$
   (b) f has a relative max at $x =$ ______ and has a relative min at $x =$



3. Find all critical points of the function. Use the First Derivative Test to determine whether f has a relative minimum, a relative maximum or neither at the critical point.

$$f(x) = x - 2\ln x, x > 0$$

4. Consider the function $f(x) = 8x^3 - 24x + 12$.
   (a) Find all critical points of f.
   (b) Find the absolute extrema of f on interval $[-3, 2]$.

5. Consider the function $g(x) = (x-2)^{2/3}$,
   (a) Find any critical points of g.
   (b) Find the absolute max and absolution min of g over $[0, 5]$.

6. For the function $k(x) = 4x^3 - 24x^2 + 36x - 20$,
   (a) Find any critical points of k.
   (b) For what x values is the function k increasing? decreasing?
   (c) Find any relative and absolute extrema of k on $[-1, 5]$.

7. For the function $h(x) = e^x - x$ defined on $[-2, 3]$,
   (a) Find any critical points of h.
   (b) For what x values is the function h increasing? decreasing?
   (c) Find any relative and absolute extrema of h.

8. Suppose $f(x)$ is continuous on $(-\infty, \infty)$ and f has two critical points at $x = -1$ and $x = 2$. If we know $f'(-2) < 0$, $f'(0) > 0$, and $f'(3) < 0$, determine whether each statement is True or False.
   (a) **T** or **F** f has a relative minimum at $x = -1$ because f is decreasing on the left side of $x = -1$ and increasing on the right side of $x = -1$.
   (b) **T** or **F** f has a relative maximum at $x = 2$ because f' is positive on the left side of $x = 2$ and negative on the right side of $x = 2$.
   (c) **T** or **F** f is decreasing on the interval $[-1, 2]$.
   (d) **T** or **F** f is decreasing on the interval $(2, \infty)$.

9.

Suppose $f(x)$ is continuous on $(-\infty, \infty)$ and f has two critical points at $x = 0$ and $x = 4$. If $f'(-1) < 0$, $f'(1) < 0$, and $f'(5) > 0$, then

(a) f has ________________ (relative minimum/relative minimum/no relative extrema) at $x = 0$.
(b) f has ________________ (relative minimum/relative minimum/no relative extrema) at $x = 4$.
(c) f is increasing on interval(s):
(d) f is decreasing on interval(s):

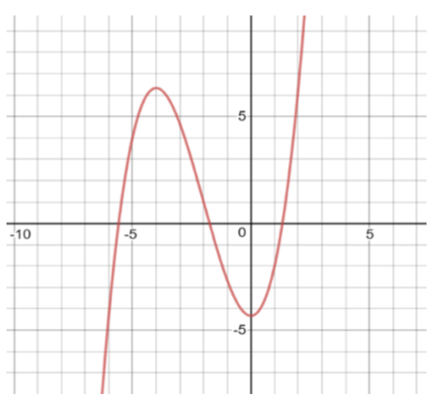
Optimization: Applications to Maximum and Minimum (Section 12.2)

1. You are running a business selling homemade bread. Your weekly revenue from the sale of q loaves bread is $R(q) = 68q - 0.1q^2$ dollars, and the weekly cost of making q loaves of bread is $C(q) = 23 + 20q$.
   (a) Find the weekly profit function $P(q)$.
   (b) Find the production level q that maximizes the weekly profit.
   (c) Find the maximum profit.
2. Suppose $C(x) = 0.02x^2 + 2x + 4000$ is the total cost for a company to produce x units of a certain product. Find the production level x that minimizes the average cost $\bar{C}(x) = \frac{C(x)}{x}$.
3. I would like to create a rectangular orchid garden that abuts my house so that the house itself forms the northern boundary. The fencing for the southern boundary costs $3 per foot, and the fencing for the east and west sides costs $5 per foot. If I have a budget of $120 for the project, what are the dimensions of the garden with the *largest area* I can enclose?
4. I need to create a rectangular vegetable patch with an area of exactly 162 square feet. The fencing for the east and west sides costs $4 per foot, and the fencing for the north and south sides costs only $2 per foot. What are the dimensions of the vegetable patch with the *least expensive* fence?
5. Worldwide annual sale of a product in 2013-2017 were projected to be approximately $q = -10p + 4220$ million units at a selling price of p dollars per unit. What selling price would have resulted in the largest projected annual revenue? What would be resulting revenue?

Higher-order Derivatives, Acceleration and Concavity (Section 12.3)

1. Find the second derivative y'' for each function.
   (a) $y = 2e^{2x-5}$
   (b) $y = \frac{7}{x} - 5\ln x$

2. The graph of a function $y = f(x)$ is given below.
   (a) $f'(-4) = f'(0) =$
   (b) Is $f''(-4)$ is positive or negative? Is $f''(0)$ is positive or negative?
   (c) If f has a point of inflection at $x = -2$, then $f''(-2) =$
   (d) f is concave________(up/down) on interval $(-\infty, -2)$, and concave________(up/down) on interval $(-2, \infty)$.



3. The graph of a function $f(x)$ is given. Fill in the blank.
   (a) The graph is concave________<u>(up/down)</u> on interval $(-\infty, 2)$, concave________<u>(up/down)</u> on interval $(2, 3)$, and concave________<u>(up/down)</u> on interval $(3, \infty)$.
   (b) The second derivative f'' is positive on: ________________and negative on: ___________________.
   (c) List the points of inflection: $x =$
   (d) Does f have any relative extrema?
   (e) Does f have any absolute extrema?

4. Suppose the position of a particle moving on a straight line is $s(t) = \sqrt{t} + 4t^2$. Find the particle's acceleration as a function of time t.

5. Let $s(t) = 4e^t - 8t^2 + 3$ be the position function of a particle moving in a straight line, where s is measured in feet and t is measured in seconds. Find its acceleration when $t = \ln 6$ seconds.

Related Rates (Section 12.5)

1. The radius of a circular puddle is growing at a rate of 15 cm/sec.
   (a) How fast is its area growing at the instant when the radius is 30 cm?
   (b) How fast is the area growing when the area is 81 square centimeters?
2. A rather flimsy spherical balloon is designed to pop at the instant its radius has reached 6 cm. Assuming the balloon is filled with helium at a rate of 13 cubic centimeters per second, calculate how fast the radius is growing at the instant it pops.
3. The average cost for the weekly manufacture of retro portable CD player is given by

$$\bar{C}(x) = 120{,}000x^{-1} + 20 + 0.0004x \text{ dollars per player,}$$

where x is the number of CD players manufactured that week. Weekly production is currently 4,000 players and is increasing at a rate of 400 players per week. What is happening to the average cost? Fill in the blank.

The average cost is ___________<u>increasing/decreasing</u> at a rate of _____dollars per player per week.

Elasticity (Section 12.6)

1. Suppose the elasticity of demand is 3.2, when the price of a product is $\$25$. This means the demand is going <u>up/down</u> by ______% for 1% increase in the price. A small increase in price will result in a <u>increase/decrease</u> in the revenue.

2. Suppose the elasticity of demand is 0.65, when the price of a product is $\$500$. This means the demand will go <u>up/down</u> by ______% for 1% increase in the price. A small increase in price will cause the revenue to <u>increase/decrease</u>.

3. The weekly sales of some backpacks is given by $q = 1080 - 18p$, where the q represents the quantity of backpacks sold at price p.
   (a) Find the elasticity of demand at the price of $20. Interpret your answer.
   (b) Is the demand at the price $20 elastic, inelastic, or unit elastic? Should the price be raised or lowered from $20 to increase the revenue?
   (c) What price will maximize the revenue?
   (d) What is the maximum weekly revenue?

4. Suppose the demand function is $q = -2p^2 + 33p$, where q represents the quantity sold at price p.
   (a) Find the price elasticity of demand $E(p)$.
   (b) Find the elasticity when $p = \$15$. If the price increase by 1%, the demand will drop by how much? Should the price be lowered or raised from $15 to increase the revenue?