Answers: MAT 210 Exam 3 Review Questions

Indefinite Integral (Section 13.1)

1.
   (a) $\frac{2}{5}x^5 + \frac{4}{x} - \frac{5}{4x^4} + 3x + C$
   (b) $7\ln|x| - \frac{1}{18x^6} + C$
   (c) $-\frac{2}{x} - \frac{10}{3}x^{\frac{3}{2}} + C$
   (d) $e^x - \frac{x^{0.7}}{0.7} + C = e^x - \frac{10}{7}x^{0.7} + C$
   (e) $\int x^2 + x - 6\,dx = \frac{x^3}{3} + \frac{x^2}{2} - 6x + C$
   (f) $\int x + 5 - \frac{2}{x}\,dx = \frac{x^2}{2} + 5x - 2\ln|x| + C$

2. $f(x) = 9e^x + 9x - 10$

3. $s(t) = \frac{1}{3}t^3 + 6t - 27$

4.
   $C(x) = 0.25x^2 + \ln|x| + 4.75$ dollars

$$C(x) = \int C'(x)\,dx = \int 0.5x + \frac{1}{x}dx = 0.25x^2 + \ln|x| + K$$

   $C(1) = 5$, So $5 = 0.25 \cdot 1^2 + \ln|1| + K$. Then solve for constant K: $K = 5 - 0.25 = 4.75$.

Substitution (Section 13.2)

1.
   (a) $16 \cdot \frac{e^{-3x}}{-3} + C = -\frac{16}{3}e^{-3x} + C$
   (b) $\frac{(5x-2)^4}{4} \cdot \frac{1}{5} + C = \frac{1}{20}(5x-2)^4 + C$
   (c) $\frac{1}{2}\ln|2x - 5| + C$
   (d) $2e^{x^2-3} + C$
   (e) $\frac{1}{22}(x^2+1)^{11} + C$
   (f) $-5(-x^2+7)^{\frac{3}{2}} + C$
   (g) $\frac{1}{10}(x^3 + x - 2)^{10} + C$
   (h) $\frac{3}{2}(\ln x)^2 + C$

Fundamental Theorem of Calculus; Definite Integral; Left Riemann Sum (Sections 13.3, 13.4)

1.

(a) $\left[6\frac{x^6}{6}+15\frac{x^5}{5}-9\frac{x^3}{3}+x\right]_0^1=[x^6+3x^5-3x^3+x]_0^1=(1+3-3+1)-(0)=2$

(b) $\left[\frac{x^2}{2}+5\ln|x|\right]_2^7=\left(\frac{7^2}{2}+5\ln 7\right)-\left(\frac{2^2}{2}+5\ln\ 2\right)=\frac{49}{2}+5\ln 7-\frac{4}{2}-5\ln 2=\frac{45}{2}+5\ln\frac{7}{2}$

(c) $\int_1^{10}\frac{1}{x^2}\,dx=\ \int_1^{10}x^{-2}\,dx=\left[\frac{x^{-1}}{-1}\right]_1^{10}=\ \left[-\frac{1}{x}\right]_1^{10}=\ -\frac{1}{10}-(-1)=\frac{9}{10}$

(d) $[-e^{-x+6}]_0^6=(-e^{-6+6})-(-e^{-0+6})=-1+e^6$

(e) $\left[5\frac{e^{3x}}{3}\right]_{-1}^1=\left[\frac{5}{3}e^{3x}\right]_{-1}^1=\frac{5}{3}e^{3(1)}-\frac{5}{3}e^{3(-1)}=\ \frac{5}{3}(e^3-e^{-3})$

(f) $[2\ln|x|]_{e^3}^{e^5}=2\ln(e^5)-2\ln(e^3)=2\,(5)-2(3)=10-6=4$

(g) $\left[\frac{e^{2x}}{2}\right]_{\ln 3}^{\ln 5}=\frac{e^{2\ln 5}-e^{2\ln 3}}{2}=\frac{e^{\ln 5^2}-e^{\ln 3^2}}{2}=\frac{25-9}{2}=\frac{16}{2}=8$

2. $\int_2^b(2x-4)\,dx=9$ >> $[x^2-4x]_2^b=9$ >> $(b^2-4b)-(2^2-4*2)=9$ >> $b^2-4b+4=9$

>> $b^2-4b-5=0$ >> $(b-5)(b+1)=0$ >> $b=5\ or\ b=\ -1\ (rejected\ since\ b>0)$. Therefore, b = 5.

3. 22.56

$\Delta x=\frac{b-a}{n}=\frac{3-1}{5}=0.4,\ x_0=a=1, x_1=x_0+\Delta x=1.4, x_2=1.8,\ \ x_3=2.2,\ \ x_4=2.6.$
LRS = $\Delta x\cdot\big(f(1)+f(1.4)+f(1.8)+f(2.2)+f(2.6)\big)=0.4(2+5.68+10.32+15.92+22.48)=22.56$

4. 0.18

$\Delta x=0.25$, LRS $=0.25\left(\frac{1}{1+2(2)}+\frac{1}{1+2(2.25)}+\frac{1}{1+2(2.5)}+\frac{1}{1+2(2.75)}\right)=0.25(0.2+0.18+0.17+0.15)=0.18$

Applications of Definite Integrals (Section 13.4)

1. Displacement $=s(6)-s(2)=\int_2^6 v(t)\,dt=\int_2^6(-t^2+8)\,dt=\left(-\frac{t^3}{3}+8t\right)_2^6$

$=\left(-\frac{6^3}{3}+8*6\right)-\left(-\frac{2^3}{3}+8*2\right)=(-72+48)-\left(-\frac{8}{3}+16\right)=\ -24+\frac{8}{3}-16\ =-\frac{112}{3}$ meters

2. Total revenue generated $=R(5000)-R(101)=\int_{101}^{5000}MR\,dx=\int_{101}^{5000}500e^{-0.001x}\,dx$

$=\left[\frac{500}{-0.001}e^{-0.001x}\right]_{101}^{5000}=\ -500\,000\,(e^{-5}-e^{-0.101})\approx 448597.54$ dollars

3. Total number of hours $= \int_2^5 f(t)\,dt = \int_2^5 (1.1t^2 - 2.6t + 2.3)\,dt = \left(\frac{1.1t^3}{3} - \frac{2.6t^2}{2} + 2.3\,t\right)_2^5$

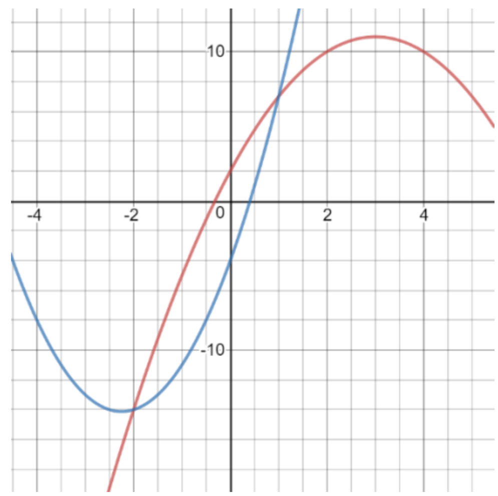
$$= \left(\frac{1.1(5)^3}{3} - \frac{2.6(5)^2}{2} + 2.3\,(5)\right) - \left(\frac{1.1(2)^3}{3} - \frac{2.6(2)^2}{2} + 2.3\,(2)\right) = \frac{45}{2} \text{ million hours of video}$$

4. Area under curve $= \int_0^{16} \sqrt{x}\,dx = \left(\frac{2}{3}x^{\frac{3}{2}}\right)_0^{16} = \left(\frac{2}{3}(16)^{\frac{3}{2}}\right) - \left(\frac{2}{3}(0)^{\frac{3}{2}}\right) = \left(\frac{2}{3}(4^2)^{\frac{3}{2}}\right) = \left(\frac{2}{3}(4^3)\right) = \frac{128}{3}$

Area between Curves (Section 14.2)

1. $\frac{27}{2}$

Details:



Find the intersection points: $-x^2 + 6x + 2 = 2x^2 + 9x - 4$

$$0 = 3x^2 + 3x - 6$$

$$0 = 3(x+2)(x-1)$$

So $x = -2$ and $x = 1$.

The area enclosed by the curves from -2 to 1 is

$\int_{-2}^{1} (\text{top} - \text{bottom})\,dx = \int_{-2}^{1} [(-x^2 + 6x + 2) - (2x^2 + 9x - 4)]\,dx$

$$= \int_{-2}^{1} (-3x^2 - 3x + 6)\,dx$$

$$= -x^3 - \frac{3}{2}x^2 + 6x|_{-2}^{1}$$

$$= (-1^3 - \frac{3}{2}(1)^2 + 6(1)) - \left(-(-2)^3 - \frac{3}{2}(-2)^2 + 6(-2)\right) = \frac{27}{2}$$

2. $\frac{32}{3}$

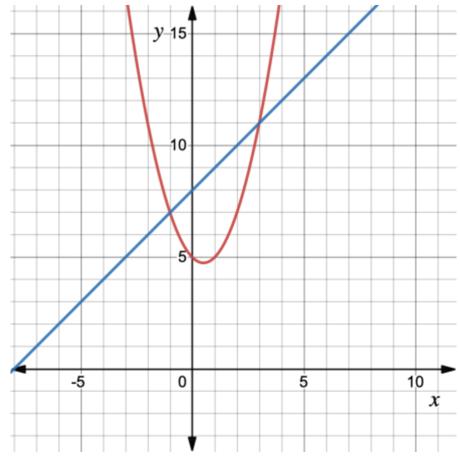
Details:

Find the intersection points: $x^2 - x + 5 = x + 8$

$$0 = x^2 - 2x - 3$$

$$0 = (x+1)(x-3)$$

So $x = 3$ and $x = -1$.



The area enclosed by the curves from -1 to 3 is

$$\int_{-1}^{3} (\text{top} - \text{bottom})\, dx = \int_{-1}^{3} [(x+8) - (x^2 - x + 5)]\, dx$$

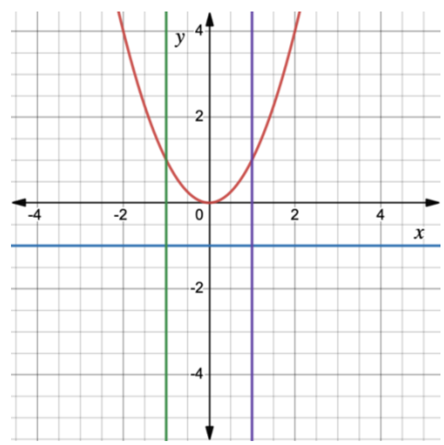
$$= \int_{-1}^{3} (-x^2 + 2x + 3)\, dx$$

$$= \frac{-x^3}{3} + x^2 + 3x \Big|_{-1}^{3}$$

$$= \left(\frac{-3^3}{3} + 3^2 + 3(3)\right) - \left(\frac{-(-1)^3}{3} + (-1)^2 + 3(-1)\right) = 9 - \left(\frac{1}{3} + 1 - 3\right) = \frac{32}{3}$$

3. $\frac{8}{3}$

Details:



The area enclosed by the curves from -1 to 1 is

$$\int_{-1}^{1} (\text{top} - \text{bottom})\, dx = \int_{-1}^{1} [x^2 - (-1)]\, dx$$

$$= \int_{-1}^{1} (x^2 + 1)\, dx$$

$$= \frac{x^3}{3} + x \Big|_{-1}^{1}$$

$$= \left(\frac{1^3}{3} + 1\right) - \left(\frac{(-1)^3}{3} + (-1)\right) = \frac{4}{3} - \left(-\frac{4}{3}\right) = \frac{8}{3}$$

4. C