Answers: MAT 210 Exam 1 Review Questions

Limits (sections 10.1, 10.3)

1. a. 22

   b. $\frac{6}{13}$

   c. 3

2. a. $\lim_{x\to\infty}\frac{3x^3-3}{-13x^3-4x^2-2}=\lim_{x\to\infty}\frac{3-\frac{3}{x^3}}{-13-\frac{4}{x}-\frac{2}{x^3}}=\frac{3}{-13}=-\frac{3}{13}$, as $x\to\infty$ the read terms aproaches 0.

   As $x\to\infty$, the numerator approaches ∞ and the denominator approaches $-\infty$. This limit expression has a $\frac{\infty}{-\infty}$ **type of indeterminate form (**Again this is not a number and not the answer.) Divide each term of the rational function by x^n, where n is the largest power of x in the expression. In this problem divide by x^3, where 3 is the largest power of x. Take the limit as $x\to\infty$, each $\frac{non-zero\ \#}{x^n}$ term approaches 0. Thus, for large x values (as $x\to\infty$ or as $x\to-\infty$), all terms of a rational function are negligible except the leading terms since the leading terms in both, in the numerator and denominator much larger compared to all the other terms. Therefore $\lim_{x\to\infty}\frac{3x^3-3}{-13x^3-4x^2-2}=\lim_{x\to\infty}\frac{3x^3}{-13x^3}=-\frac{3}{13}$. In general, if the degree of the numerator of a rational function is the same as the degree of the denominator then the limit will be the ratio of the leading terms as $x\to\infty$ or as $x\to-\infty$.

   b. $\lim_{x\to\infty}\frac{7x^2-5}{19x^3-3x-7}=\lim_{x\to\infty}\frac{7x^2}{19x^3}=\lim_{x\to\infty}\frac{7}{19x}=0$.

   As $x\to\infty$, the numerator approaches ∞ and the denominator approaches ∞. This limit expression has a $\frac{\infty}{\infty}$ **type of indeterminate form.** Again this is not a number and not the answer. In general, if the degree of the numerator of a rational function is smaller than the degree of the denominator then the limit is 0 as $x\to\infty$ or as $x\to-\infty$.

   c. $\lim_{x\to\infty}\frac{-9x^5+5}{12x^3-3x-7}=\lim_{x\to\infty}\frac{-9x^5}{12x^3}=\lim_{x\to\infty}\frac{-9x^2}{12}=-\infty$.

   As $x\to\infty$, the numerator approaches $-\infty$ and the denominator approaches ∞. This limit expression has a $\frac{-\infty}{\infty}$ **type of indeterminate form.** In general, if the degree of the numerator of a rational function is greater than the degree of the denominator then the limit does not exist. The limit is either ∞ or $-\infty$ depending on the sign of the leading coefficients.

   All the limit expressions discussed above have $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$ indeterminate form, so there is no general rule to find the limit, each individual problem requires further analysis to determine the limit.

d. $\lim_{x\to\infty} \frac{-3}{11x^3} = 0$

As $x \to \infty$, the denominator approaches ∞. This limit expression has a $\frac{\boldsymbol{non-zero\ \#}}{\infty}$ **type of determinate form,** thus the limit is 0. A non-zero number divided by a large number results a number a number that is close to 0. As the dividend increases without bound the value of the fraction is getting arbitrarily close to 0.

e. $\lim_{x\to\infty} \frac{-3x^3}{11} = -\infty$

As $x \to \infty$, the numerator approaches $-\infty$. This limit expression has a $\frac{-\infty}{\boldsymbol{non-zero\ \#}}$ **type of determinate form,** thus the limit is $-\infty$.

3. 2

4. 0

5. 2

6. ∞

7. 0

8. $\lim_{t\to+\infty} m(t) = 0$. In the long term, the amount of drug in the blood will completely disappear.

9. $\lim_{t\to+\infty} p(t) = 500$. The population of squirrels will approach 500 as the time increases (in the long run).

10.

a) $\lim_{t\to+\infty} W(t) = \infty$. In the long term, the popularity of Twitter among social media sites will increase without bound.

b) $\lim_{t\to+\infty} \frac{W(t)}{L(t)} = 8.25$. In the long term, the popularity of Twitter will increase by 8.25 more than LinkedIn.

Note: A percentage can't rise beyond 100, so extrapolating the models to obtain long term predictions gives meaningless results.

Rates of Change (sections 10.4, 10.5)

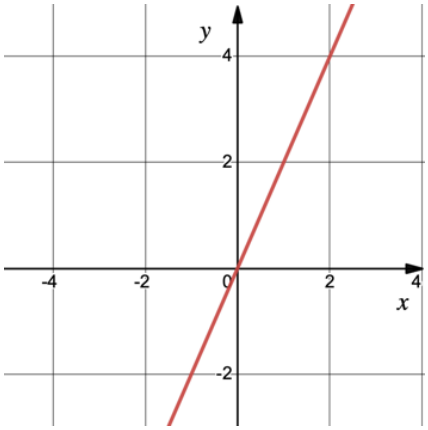
1. 21
2. - 2 dollars/day
3. 0.3125 million barrels per year.

U.S. daily oil imports from a certain country increased by an average rate of 0.3125 million barrels per year over the period 1991 to 1999.

4.

a) The sign of f' is negative on the interval $(-\infty, 0)$, it is 0 at $x = 0$, then positive on the interval $(0, \infty)$

b) f' is always increasing



5.

a) 425/9 beetles per week

b)

A) F
B) T
C) T
D) F
E) T
F) F
G) T
H) F
I) T

6.

A. In 1998 the number of students who took the AP Calculus exam is 156,682 and this number is increasing at a rate of 9,644 student per year.

Differentiation (Power Rule, Product Rule, Quotient Rule, Contant Multiple Rule) **(sections 11.1, 11.3)**

1.

a) $f'(x) = 8x$

b) $f'(x) = -2$

c) $f'(x) = 4x + 6$

d) $f'(x) = \frac{9}{x^5}$

e) $f'(x) = -4x^{-3/2} + 4x^{-2}$

f) $g'(x) = 6x + 8 + 12x^{-7} - 11.5x^{1.3}$

g) $h'(x) = 72x^2 - 32x + 72$

h) $k'(x) = -0.5x^{-2}$

i) $m'(x) = 70x - 21x^{-4} + 49x^{-2}$

j) $v'(x) = \frac{20}{(4x+8)^2}$

k) $p'(x) = \frac{2x\cdot(x^3-2x)-(x^2+1)\cdot(3x^2-2)}{(x^3-2x)^2} = \frac{-x^4-5x^2+2}{(x^3-2x)^2}$

Applications to Derivatives and Rates of Change

Tangent Lines

1. $y = 8x - 6$
2. $y = 6x - 11$
3. $f'(1) = 17$

Average Velocity and Instantaneous Velocity

1.
   a) 2 m/s
   b) 3 m/s

Marginal Analysis (section 11.2)

1.

   a) $MP(x) = 8 - \frac{1}{\sqrt{x}}$

   b) 7.8 dollars per box. After 25 boxes of cookies have been sold, the total profit will increase by about 7.8 dollars per additional box sold, or the profit from selling the 26th box is about 7.8 dollars.

2. $MC(x) = 16x \quad MR(x) = 12x^2 + 2 \quad MP(x) = 12x^2 - 16x + 2$
3. profit = 12798 dollars Marginal profit = 506 dollars per book

4.

a) $P(x) = -150 + 6.9x - 0.002x^2$

b) 520 dollars

c) $MP(x) = 6.9 - 0.004x$

d) 6.5 dollars per candle. The profit from selling the 101st candle is about 6.5 dollars. Or the total profit will increase by 6.5 dollars per candle sold, after 100 candles are sold.