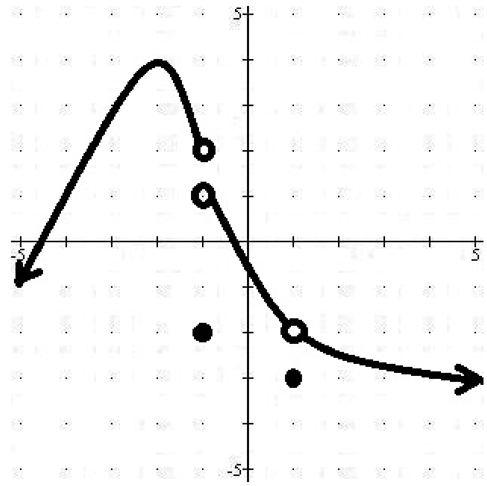
MAT 251 – Exam 1 Review

(Problems 1 – 10) Use the graph of the function f(x) to the right to find the following. Write "DNE" if the limit doesn't exist.




1) $\lim_{x\to -1^+} f(x)$ 2) $\lim_{x\to 1} f(x)$ 3) $\lim_{x\to -2} f(x)$

4) $\lim_{x\to 1^-} f(x)$ 5) $f(-1)$ 6) $f(1)$

7) $\lim_{x\to -1^-} f(x)$ 8) $\lim_{x\to 1^+} f(x)$ 9) $\lim_{x\to -1} f(x)$

10) List all values of x at which f(x) has a discontinuity.

Find the following limits. Write "DNE" if the limit doesn't exist.
Find the exact answer (No Decimal Approximations).

11) $\lim_{x\to 3}\frac{x^2-4x+3}{x-3}$ 12) $\lim_{x\to \frac{\pi}{4}}\frac{2x\sin x}{x^2}$ 13) $\lim_{x\to 2}\frac{x^2-9}{x-3}$ 14) $\lim_{x\to 0}\frac{1-\cos x}{3x}$

15) $\lim_{x\to 2}(3x^2+2x-16)$ 16) $\lim_{h\to 0}(3x^2+2xh+h^2)$ 17) $\lim_{x\to 2}\frac{1}{x-2}$

18) $\lim_{x\to 2}\frac{3x^2+5x-2}{x^2-3x-10}$ 19) $\lim_{x\to 3}\frac{x^2+5x+6}{x^2-x-6}$

| t | F(t) |
|---|---|
| 0 | 0 |
| 1 | 12 |
| 2 | 16 |
| 3 | 18 |
| 4 | 14 |

Use the chart to the right to answer the following. Give exact answers.

20) Find the average rate of change of F(t) from t = 1 to t = 3.

21) Find the average rate of change of F(t) from t = 2 to t = 4.

A car's position t seconds after it starts moving is given by $s(t)=6t\sqrt{t}$ where s(t) is in feet. The average rate of change of distance with respect to time is called the average velocity.

22) Find the average velocity from t = 0 to t = 4. (This would be in feet per second.)

23) Find the average velocity from t = 4 to t = 9.

For each of the following, find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$.

24) $f(x)=2x^2-3x+4$ 25) $f(x)=3\sqrt{x+5}$

26) $f(x)=\frac{3}{2x+1}$ 27) $f(x)=5x-4$

Find the derivative of each of the functions below. SIMPLIFY your answers.

28) $f(x) = 2x^5 - 3x^2 + 4$

29) $g(x) = \dfrac{3x^3 - 4x + 8}{2x}$

30) $h(x) = \sqrt[4]{x^9} + 7$

31) $f(x) = \dfrac{2x - 3}{4x + 5}$ (Using the Quotient Rule)

32) $f(x) = 5x^3 \sin x$

33) $f(x) = \dfrac{\cos x}{x^2}$ (Using the Quotient Rule)

34) $g(x) = \dfrac{1 - \cos x}{\sin x}$ (Using the Quotient Rule)

35) $f(x) = \sec^2 x$

The height (in feet) of a ball t seconds after it is thrown upwards from the roof of a building is given by $h(t) = -16t^2 + 40t + 100$

36) Find the height of the ball after 3 seconds. Include the units with your answer.

37) Find the velocity of the ball after 3 seconds. Include the units with your answer.

38) Find the acceleration of the ball after 3 seconds. Include the units with your answer.

Find the derivative of each of the functions below. SIMPLIFY your answers.

39) $f(x) = (5x - 8)^{40}$

40) $g(x) = \tan(x^2 - 9)$

41) $h(x) = \sqrt{10 - x^3}$

42) $f(x) = \left(\dfrac{x - 1}{x + 1}\right)^5$

43) $f(x) = \cot(\cos x)$

44) $f(x) = \cot^3 x$

45) In a small town, the number of people that are playing a new game t weeks after it is introduced is given by $g(t) = \dfrac{800t}{t^2 + 8}$. Find the instantaneous rate of change in the number of people playing the game after exactly four weeks and interpret the answer. (Round to the nearest unit.)

Find the specified Higher-Order Derivatives

46) $y = 2t^5 - 3t^3 + 5t + 12$ find $\dfrac{d^3y}{dt^3}$

47) $f(x) = \tan(2x)$ find $f''(x)$

48) $y = \dfrac{2x - 3}{x + 4}$ A) find $\dfrac{d^2y}{dx^2}$ B) find $\left.\dfrac{d^2y}{dx^2}\right|_{x=1}$

49) $g(x) = \dfrac{4}{2x + 3}$ A) find $g^{(4)}(x)$ B) find $g^{(4)}(-2)$

50) Find the equation of the line tangent to $f(x) = 3x^2 - 5x + 8$ at x = 2. Answer in slope-intercept form.

51) Find the equation of the line tangent to $f(x) = 4\sqrt{x + 5}$ at x = 4. Answer in slope-intercept form.

Solutions

1) 1 2) -2 3) 4 4) -2 5) -2 6) -3 7) 2 8) -2 9) DNE

10) f(x) is discontinuous at x = -1 and also at x = 1

11) 2 12) $\frac{4\sqrt{2}}{\pi}$ 13) 5 14) 0 (Try numerical approach) 15) 0 16) $3x^2$ 17) DNE

18) $-\frac{5}{3}$ 19) DNE 20) 3 21) -1 22) 12 feet per second 23) 22.8 feet per second

24) 4x + 2h – 3 25) $\frac{3}{\sqrt{x+h+5}+\sqrt{x+5}}$ 26) $\frac{-6}{(2x+1)(2x+2h+1)}$ 27) 5

28) $10x^4 - 6x$ 29) $3x - 4x^{-2}$ (Hint: "Break the fraction up") 30) $\frac{9}{4}x^{\frac{5}{4}}$ 31) $\frac{22}{(4x+5)^2}$

32) $5x^3 \cos x + 15x^2 \sin x$ 33) $\frac{-x\sin x - 2\cos x}{x^3}$ 34) $\frac{1-\cos x}{\sin^2 x}$ 35) $2\sec^2 x \tan x$

36) 76 ft 37) $-56\frac{ft}{sec}$ 38) $-32\frac{ft}{sec^2}$ 39) $200(5x-8)^{39}$ 40) $2x\sec^2(x^2-9)$

41) $\frac{-3x^2}{2\sqrt{10-x^3}}$ 42) $\frac{10(x-1)^4}{(x+1)^6}$ 43) $\sin x(\csc^2(\cos x))$ 44) $-3\cot^2 x \csc^2 x$

45) $-\frac{100}{9} \approx -11$ Interpretation: The number of people playing the game is decreasing at a rate of about 11 people per week exactly 4 weeks after it is introduced.

46) $120t^2 - 18$ 47) $8\sec^2(2x)\tan(2x)$ 48A) $\frac{-22}{(x+4)^3}$ B) $\frac{-22}{125}$

49A) $\frac{1536}{(2x+3)^5}$ B) -1536 50) y = 7x – 4 51) $y = \frac{2}{3}x + \frac{28}{3}$