MAT 210 Exam 3 Review Questions

Indefinite Integral (Section 13.1)

1. Find the indefinite integrals.
   (a) \( \int 2x^4 - 4x^{-2} + 5x^{-5} + 3 \, dx \)
   (b) \( \int \frac{2}{x} + \frac{1}{3x^7} \, dx \)
   (c) \( \int \frac{2}{x^2} - 5\sqrt{x} \, dx \)
   (d) \( \int e^x - x^{-0.3} \, dx \)
   (e) \( \int (x + 3)(x - 2) \, dx \)
   (f) \( \int \frac{x^2 + 5x - 2}{x} \, dx \)

   Answer:
   (a) \( \frac{2}{5}x^5 + \frac{4}{x} - \frac{5}{4x^4} + 3x + C \)
   (b) \( 7\ln|x| - \frac{1}{18x^6} + C \)
   (c) \( -\frac{2}{x} - \frac{10}{3} \frac{2}{x^3} + C \)
   (d) \( e^x - \frac{x^{0.7}}{0.7} + C = e^x - \frac{10}{7}x^{0.7} + C \)
   (e) \( \int x^2 + x - 6 \, dx = \frac{x^3}{3} + \frac{x^2}{2} - 6x + C \)
   (f) \( \int x + 5 - \frac{2}{x} \, dx = \frac{x^2}{2} + 5x - 2\ln|x| + C \)

2. Find \( f(x) \) if \( f(0) = -1 \) and the derivative \( f'(x) = 9e^x + 9 \).

   Answer: \( f(x) = 9e^x + 9x - 10 \)

3. The velocity of a particle moving in a straight line is \( v(t) = t^2 + 6 \). Find the expression for the position, \( s(t) \), of the particle at time \( t \), if \( s(3) = 0 \).

   Answer: \( s(t) = \frac{1}{3}t^3 + 6t - 27 \)

4. Suppose the function \( C(x) \) gives the total cost (in dollars) of producing \( x \) units of a certain product. The marginal cost of producing the \( x \)th unit is \( C'(x) = 0.5x + \frac{1}{x} \). If the cost to produce the first unit is 5 dollars, find the cost function \( C(x) \).

   Answer: \( C(x) = 0.25x^2 + \ln|x| + 4.75 \text{ dollars} \)

   \[ C(x) = \int C'(x) \, dx = \int (0.5x + \frac{1}{x}) \, dx = 0.25x^2 + \ln|x| + K \]

   \( C(1) = 5 \), so \( 5 = 0.25 \cdot 1^2 + \ln|1| + K \). Then solve for constant \( K \): \( K = 5 - 0.25 = 4.75 \).
Substitution (Section 13.2)

5. Use integration by substitution to find the integrals.
   (a) \( \int 16e^{-3x} \, dx \) (can also use short-cut formula)
   (b) \( \int (5x - 2)^3 \, dx \) (can also use short-cut formula)
   (c) \( \int \frac{1}{2x-5} \, dx \) (can also use short-cut formula)
   (d) \( \int 4xe^{x^2-3} \, dx \)
   (e) \( \int x(x^2 + 1)^{10} \, dx \)
   (f) \( \int 15x\sqrt{x^2 + 7} \, dx \)
   (g) \( \int (3x^2 + 1)(x^3 + x - 2)^9 \, dx \)

**Answer:**

(a) \( 16 \cdot e^{-3x} + C = -\frac{16}{3} e^{-3x} + C \)
(b) \( \frac{(5x-2)^4}{4} \cdot \frac{1}{5} + C = \frac{1}{20} (5x - 2)^4 + C \)
(c) \( \frac{1}{2} \ln|2x - 5| + C \)
(d) \( 2e^{x^2-3} + C \)
(e) \( \frac{1}{22} (x^2 + 1)^{11} + C \)
(f) \( -5(-x^2 + 7)^{3/2} + C \)
(g) \( \frac{1}{10} (x^3 + x - 2)^{10} + C \)

Fundamental Theorem of Calculus; Definite Integral; Left Riemann Sum (Sections 13.3, 13.4)

6. Evaluate the definite integrals.
   (a) \( \int_0^1 (6x^5 + 15x^4 - 9x^2 + 1) \, dx \)
   (b) \( \int_{\frac{1}{2}}^{2} \left( x + \frac{5}{x} \right) \, dx \)
   (c) \( \int_{\frac{1}{3}}^{10} \frac{1}{x^2} \, dx \)
   (d) \( \int_0^6 e^{-x+6} \, dx \)
   (e) \( \int_{-1}^{1} 5e^{3x} \, dx \)
   (f) \( \int_{e^{-5/2}}^{e^{5/2}} \frac{1}{x} \, dx \)
   (g) \( \int_{\ln 3}^{\ln 5} e^{2x} \, dx \)

**Answer:**

(a) \( \frac{1}{2} \)
(b) \( \frac{45}{2} + 5 \ln \left( \frac{7}{2} \right) \)
(c) \( \frac{9}{10} \)
(d) \(-1 + e^6\)
(e) \(\frac{5}{3}(e^3 - e^{-3})\)
(f) 4
(g) 8

7. Assume that \( b \) is a positive number, solve the following equation for \( b \).

\[ \int_2^b (2x - 4) \, dx = 9 \]

**Answer:** \( b = 5 \)

8. Calculate the left Riemann sum for the function \( f(x) = 3x^2 + 2x - 3 \) over the interval \([1, 3]\), with \( n = 5 \).

**Answer:** 22.56

\[ \Delta x = \frac{b-a}{n} = \frac{3-1}{5} = 0.4, \quad x_0 = a = 1, \quad x_1 = x_0 + \Delta x = 1.4, \quad x_2 = 1.8, \quad x_3 = 2.2, \quad x_4 = 2.6. \]

\[ \text{LRS} = \Delta x \cdot \left( f(1) + f(1.4) + f(1.8) + f(2.2) + f(2.6) \right) = 0.4(2 + 5.68 + 10.32 + 15.92 + 22.48) = 22.56 \]

9. Use a left Riemann sum to estimate the definite integral with \( n = 4 \) subintervals.

\[ \int_2^3 \frac{1}{1+2x} \, dx \]

**Answer:** 0.18

\[ \Delta x = 0.25, \quad \text{LRS} = 0.25 \left( \frac{1}{1+2(2)} + \frac{1}{1+2(2.25)} + \frac{1}{1+2(2.5)} + \frac{1}{1+2(2.75)} \right) = 0.25(0.2 + 0.18 + 0.17 + 0.15) = 0.18 \]

Applications of Definite Integrals (Section 13.4)

10. A particle moves in a straight line with velocity \( v(t) = -t^2 + 8 \) meters per second, where \( t \) is time in seconds. Find the displacement of the particle between \( t = 2 \) and \( t = 6 \) seconds.

**Answer:** \(-37 \) meters

\[ \text{Displacement} = s(6) - s(2) = \int_2^6 v(t) \, dt = \int_2^6 (-t^2 + 8) \, dt = -\frac{112}{3} \approx -37 \text{ meters.} \]

11. The marginal revenue of the \( x \)th box of flash cards sold is \( 500e^{-0.001x} \) dollars. Find the revenue generated by selling box 101 through 5,000.

**Answer:** 448,598 dollars

\[ \text{Total revenue generated} = R(5000) - R(101) = \int_{101}^{5000} MR \, dx = \int_{101}^{5000} 500e^{-0.001x} \, dx \approx 448597.54 \text{ dollars} \]
12. Since YouTube first became available to the public in mid-2005, the rate at which video has been uploaded to this site can be approximated by \( f(t) = 1.1t^2 - 2.6t + 2.3 \) million hours of videos per year \((0 \leq t \leq 9)\), where \( t \) is time in years since June 2005. Use a definite integral to estimate the total number of hours of video uploaded from June 2007 to June 2010.

**Answer:** 23 million hours of video

Total number of hours \( = \int_2^5 f(t) \, dt = \int_2^5 (1.1t^2 - 2.6t + 2.3) \, dt \approx 23 \) million hours of video

13. Calculate the area of the region bounded by \( y = \sqrt{x} \), the \( x \)-axis, and the lines \( x = 0 \) and \( x = 16 \).

**Answer:** \( \frac{128}{3} \)

Area under curve \( = \int_0^{16} \sqrt{x} \, dx = \frac{128}{3} \)

**Area between Curves (Section 14.2)**

14. Find the area of the region enclosed by the curves of \( y = -x^2 + 6x + 2 \) and \( y = 2x^2 + 9x - 4 \).

**Answer:** 13.5

Find the intersection points: \(-x^2 + 6x + 2 = 2x^2 + 9x - 4\)

\[ 0 = 3x^2 + 3x - 6 \]

\[ 0 = 3(x + 2)(x - 1) \]

So \( x = -2 \) and \( x = 1 \).

The area enclosed by the curves from \(-2\) to \(1\) is

\[
\int_{-2}^{1} (\text{top} - \text{bottom}) \, dx = \int_{-2}^{1} [(-x^2 + 6x + 2) - (2x^2 + 9x - 4)] \, dx
\]

\[
= \int_{-2}^{1} (-3x^2 - 3x + 6) \, dx
\]

\[
= -x^3 - \frac{3}{2}x^2 + 6x \bigg|_{-2}^{1}
\]

\[
= (-1^3 - \frac{3}{2}1^2 + 6(1)) - \left(-(-2)^3 - \frac{3}{2}(-2)^2 + 6(-2)\right) = 13.5
\]

15. Find the area of the region enclosed by the curves of \( f(x) = x^2 - x + 5 \) and \( g(x) = x + 8 \).

**Answer:** \( \frac{32}{3} \)

16. Find the area of the region between \( y = x^2 \) and \( y = -1 \) from \( x = -1 \) and \( x = 1 \).

**Answer:** \( \frac{8}{3} \)
17. Which of the following calculates the area of the region(s) between the curves $y = x^2$ and $y = 1$ from $x = -1$ to $x = 2$?

A. $\int_{-1}^{2}(x^2 - 1)dx$
B. $\int_{-1}^{2}(1 - x^2)dx$
C. $\int_{-1}^{1}(1 - x^2)dx + \int_{1}^{2}(x^2 - 1)dx$
D. $\int_{-1}^{1}(x^2 - 1)dx + \int_{1}^{2}(1 - x^2)dx$
E. None of the above.

Answer: C

**Average Value (Section 14.3)**

18. Find the average value of $f(x) = 6e^{0.5x}$ over the interval $[-1, 3]$.

Answer: $3(e^{1.5} - e^{-0.5})$

The average value of a continuous function $f(x)$ over interval $[a, b]$ is $\frac{1}{b-a}\int_{a}^{b} f(x) \, dx$.

$$\frac{1}{3 - (-1)} \int_{-1}^{3} 6e^{0.5x} \, dx = \frac{1}{4} \cdot 6 \cdot \frac{e^{0.5x}}{0.5} \bigg|_{-1}^{3} = 3(e^{1.5} - e^{-0.5})$$

19. Find the average of the function $f(x) = x^3 - x$ over the interval $[0, 2]$.

Answer: 1

20. Find the average value of the function $f(x) = 6x^2 - 4x + 7$ over the interval $[-2, 2]$.

Answer: 15