MAT 210 Exam 2 Review Questions

Chain Rule (section 11.4)

1. Find the derivative y' of each function.
   (a) $y = 0.4(3x^2 + 2x - 8)^5$
   (b) $y = \sqrt{x^3 - 50x}$
   (c) $y = (6x + 1)^{4/3}$
   (d) $y = \frac{25}{(x^2+x+2)^4}$
   (e) $y = 8e^{-2x}$

   ***Answer*:**

   (a) $2(3x^2 + 2x - 8)^4(6x + 2)$

   (b) $\frac{1}{2}(x^3 - 50x)^{-\frac{1}{2}} \cdot (3x^2 - 50) = \frac{3x^2-50}{2\sqrt{x^3-50x}}$

   (c) $\frac{4}{3}(6x + 1)^{1/3} \cdot 6 = 8(6x + 1)^{\frac{1}{3}} = 8\sqrt[3]{6x + 1}$

   (d) $25(-4)(x^2 + x + 2)^{-5} \cdot (2x + 1) = -100(2x + 1)(x^2 + x + 2)^{-5} = -\frac{100(2x+1)}{(x^2+x+2)^5}$

   (e) $-16e^{-2x}$

Derivative of Logarithmic and Exponential Functions (section 11.5)

2. Find the derivative y' of each function.
   (a) $y = 9\ln(2x)$
   (b) $y = \ln(5x^3 + x^2 + 4)$
   (c) $y = \ln|x^3 - 8x|$
   (d) $y = x - x\ln x$
   (e) $y = 4e^{x^5-3x}$
   (f) $y = (x^2 - 2x)e^{2x+3}$
   (g) $y = e^{5/x}$

   ***Answer*:**

   (a) $9 \cdot \frac{1}{2x} \cdot 2 = \frac{9}{x}$

   (b) $\frac{15x^2+2x}{5x^3+x^2+4}$

(c) $\frac{3x^2-8}{x^3-8x}$

(d) $1-\left(1\cdot\ln x+x\cdot\frac{1}{x}\right)=-\ln x$

(e) $4e^{x^5-3x}\cdot(5x^4-3)$

(f) $(2x-2)e^{2x+3}+(x^2-2x)e^{2x+3}\cdot 2=(2x^2-2x-2)e^{2x+3}$

(g) $e^{5/x}\cdot(-5x^{-2})$

Implicit Differentiation (section 11.6)

3. Find the derivative $\frac{dy}{dx}$.

(a) $x^3-y^3+y=3$

(b) $6x^2y-15x=y^2$

(c) $xe^y-e^x=0$

***Answer*:**

(a) $\frac{dy}{dx}=\frac{3x^2}{3y^2-1}$

(b) $\frac{dy}{dx}=\frac{15-12xy}{6x^2-2y}$

(c) $\frac{dy}{dx}=\frac{e^x-e^y}{xe^y}$

Maxima and Minima (section 12.1)

4. The function of a function f on $[-3,3]$ is given below.

(a) f has a relative minimum at $x=$

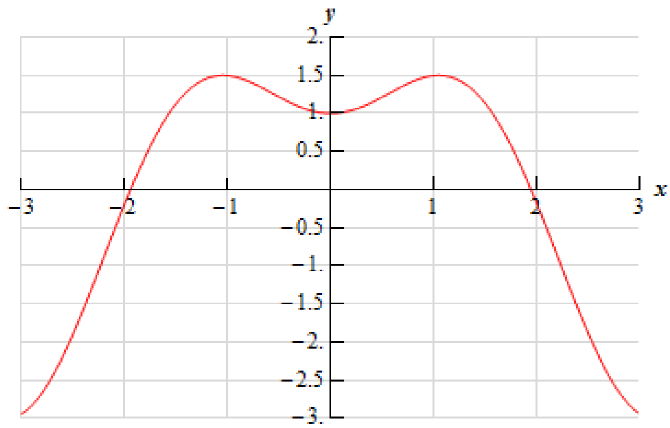
(b) f has an absolute maximum at $x=$

(c) f has an absolute minimum at $x=$

(d) f' is zero at $x=$

(e) f' is positive on interval(s):

(f) f' is negative on interval(s):



***Answer*:**

(a) 0

(b) $-1, 1$

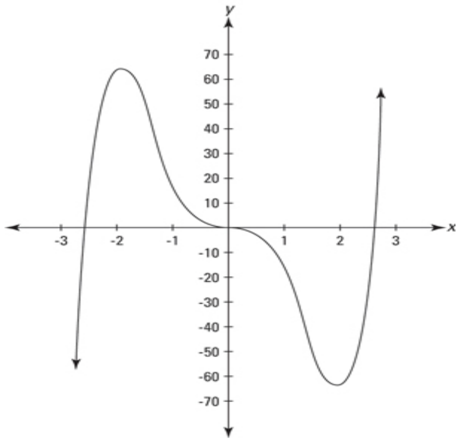
(c) $-3, 3$

(d) $-1, 0, 1$

(e) $(-3,-1)$ and $(0,1)$

(f) $(-1,0)$ and $(1,3)$

5. The graph of a function f is given below.
   (a) f' is zero at $x =$
   (b) f has a relative max at $x =$ _____ and has a relative min at $x =$



***Answer*:**

(a) $-2, 0, 2$
(b) $-2; 2$

6. Find all critical points of the function. Use the First Derivative Test to determine whether f has a relative minimum, a relative maximum or neither at the critical point.

$$f(x) = x - 2\ln x, x > 0$$

***Answer*:** Critical point $x = 2$; f has a relative minimum at $x = 2$, which is equal to $f(2) = 2 - 2\ln 2$.

7. Consider the function $f(x) = 8x^3 - 24x + 12$.
   (a) Find all critical points of f.
   (b) Find the absolute extrema of f on interval $[-3, 2]$.

***Answer*:**
(a) Critical points: $x = -1, 1$.
(b) Abs Max $= f(-1) = f(2) = 28$; Abs Min$= f(-3) = -132$.

8. Consider the function $g(x) = (x - 2)^{2/3}$,
   (a) Find any critical points of g.
   (b) Find the absolute max and absolution min of g over $[0, 5]$.

***Answer*:**

(a) Critical point: $x = 2$
(b) Abs Max $= g(5) = 3^{2/3}$; Abs Min $= g(2) = 0$.

9. For the function $k(x) = 4x^3 - 24x^2 + 36x - 20$,
   (a) Find any critical points of k.
   (b) For what x values is the function k increasing? decreasing?
   (c) Find any relative and absolute extrema of k on $[-1, 5]$.

***Answer*:**

(a) Critical points: $x = 1, 3$.
(b) Increasing on $(-\infty, 1) \cup (3, \infty)$; decreasing on $(1, 3)$.
(c) Abs Max $= k(5) = 60$; Abs Min $= k(-1) = -84$; Rel Max $= k(1) = -4$; Rel Min $= k(3) = -20$.

10. For the function $h(x) = e^x - x$ defined on $[-2, 3]$,
(a) Find any critical points of h.
(b) For what x values is the function h increasing? decreasing?
(c) Find any relative and absolute extrema of h.

***Answer*:**

(a) Critical points: $x = 0$.
(b) Increasing on $(0, 3)$; decreasing on $(-2, 0)$.
(c) Abs Max $= h(3) = e^3 - 3$; Abs Min $= h(0) = 1$; Rel Max $= h(-2) = e^{-2} + 2$; Rel Min: None.

11. Suppose $f(x)$ is continuous on $(-\infty, \infty)$ and f has two critical points at $x = -1$ and $x = 2$. If we know $f'(-2) < 0$, $f'(0) > 0$, and $f'(3) < 0$, determine whether each statement is True or False.
(a) **T** or **F** f has a relative minimum at $x = -1$ because f is decreasing on the left side of $x = -1$ and increasing on the right side of $x = -1$.
(b) **T** or **F** f has a relative maximum at $x = 2$ because f' is positive on the left side of $x = 2$ and negative on the right side of $x = 2$.
(c) **T** or **F** f is decreasing on the interval $[-1, 2]$.
(d) **T** or **F** f is decreasing on the interval $(2, \infty)$.

***Answer*: T, T, F, T.**

12. Suppose $f(x)$ is continuous on $(-\infty, \infty)$ and f has two critical points at $x = 0$ and $x = 4$. If $f'(-1) < 0$, $f'(1) < 0$, and $f'(5) > 0$, then
(a) f has ________________ (relative minimum/relative minimum/no relative extrema) at $x = 0$.
(b) f has ________________ (relative minimum/relative minimum/no relative extrema) at $x = 4$.
(c) f is increasing on interval(s):
(d) f is decreasing on interval(s):

***Answer*:**

(a) No relative extrema
(b) Relative min
(c) $(4, \infty)$
(d) $(-\infty, 4)$

Optimization: Applications to Maximum and Minimum (Section 12.2)

13. You are running a business selling homemade bread. Your weekly revenue from the sale of q loaves bread is $R(q) = 68q - 0.1q^2$ dollars, and the weekly cost of making q loaves of bread is $C(q) = 23 + 20q$.
(a) Find the weekly profit function $P(q)$.
(b) Find the production level q that maximizes the weekly profit.
(c) Find the maximum profit.

***Answer*:**

(a) $P(q) = 68q - 0.1q^2 - (23 + 20q) = -0.1q^2 + 48q - 23$
(b) Set $P' = -0.2q + 48 = 0$, and solve for q: $q = 240$
(c) $P(240) = -0.1 \cdot 240^2 + 48 \cdot 240 - 23 = 5737$ dollars.

14. Suppose $C(x) = 0.02x^2 + 2x + 4000$ is the total cost for a company to produce x units of a certain product. Find the production level x that minimizes the average cost $\bar{C}(x) = \frac{C(x)}{x}$.

    ***Answer*:** $\bar{C}(x) = 0.02x + 2 + 4000x^{-1}$. Set $\bar{C}'(x) = 0.02 - 4000x^{-2} = 0$ and solve for x: $x = 447$ units.

15. I would like to create a rectangular orchid garden that abuts my house so that the house itself forms the northern boundary. The fencing for the southern boundary costs \$3 per foot, and the fencing for the east and west sides costs \$5 per foot. If I have a budget of \$120 for the project, what are the dimensions of the garden with the *largest area* I can enclose?

    ***Answer*:**

    Maximize area $A = xy$ subject to cost $3x + 10y = 120$, where x is the length of the north and south sides, and y is the length of east and west sides.

    Dimension is $x = 20\ ft, y = 6\ ft$. Max area = 120 square feet.

16. I need to create a rectangular vegetable patch with an area of exactly 162 square feet. The fencing for the east and west sides costs \$4 per foot, and the fencing for the north and south sides costs only \$2 per foot. What are the dimensions of the vegetable patch with the *least expensive* fence?

    ***Answer*:**

    Minimize the cost $C = 4x + 8y$ subject to area $xy = 162$, where x is the length of the north and south sides, and y is the length of east and west sides.

    Dimension is $x = 18\ ft, y = 9\ ft$. Minimum cost = 144 dollars.

17. Worldwide annual sale of a product in 2013-2017 were projected to be approximately $q = -10p + 4220$ million units at a selling price of p dollars per unit. What selling price would have resulted in the largest projected annual revenue? What would be resulting revenue?

    ***Answer*:** $R(p) = p \cdot q = p(-10p + 4220) = -10p^2 + 4220p$. Set $R'(p) = -20p + 4220 = 0$ and solve for p: $p = \$\ 211$. Maximum revenue is $R(211) = 211 \cdot 2110 = \$445210$.

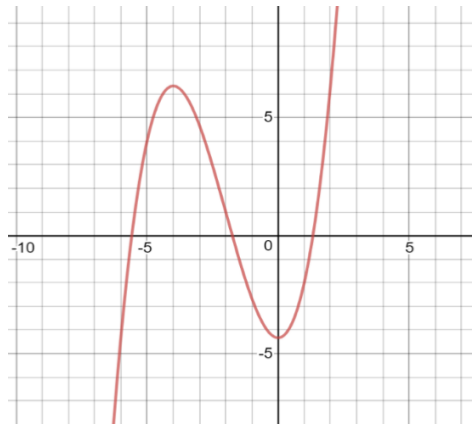
Higher-order Derivatives, Acceleration and Concavity (Section 12.3)

18. Find the second derivative y'' for each function.
    (a) $y = 2e^{2x-5}$
    (b) $y = \frac{7}{x} - 5\ln x$

***Answer*:**

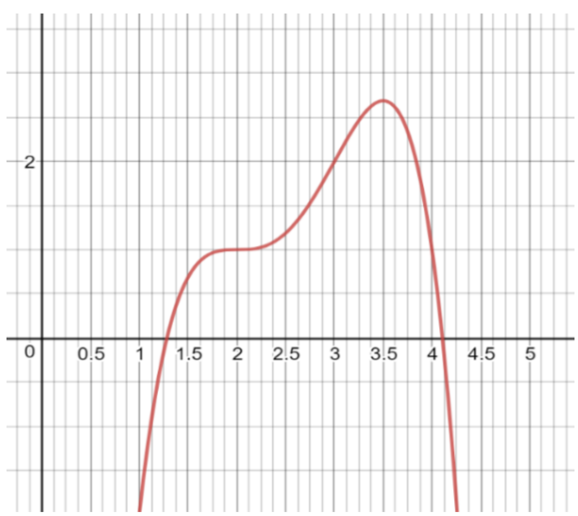
(a) $y'' = 8e^{2x-5}$
(b) $y'' = 14x^{-3} + 5x^{-2}$

19. The graph of a function $y = f(x)$ is given below.
   (a) $f'(-4) = f'(0) =$
   (b) Is $f''(-4)$ is positive or negative? Is $f''(0)$ is positive or negative?
   (c) If f has a point of inflection at $x = -2$, then $f''(-2) =$
   (d) f is concave________(up/down) on interval $(-\infty, -2)$, and concave________(up/down) on interval $(-2, \infty)$.



***Answer*:** (a) 0; (b) negative, positive; (c) 0; (d) down, up.

20. The graph of a function $f(x)$ is given. Fill in the blank.
   (a) The graph is concave________(up/down) on interval $(-\infty, 2)$, concave________(up/down) on interval $(2, 3)$, and concave________(up/down) on interval $(3, \infty)$.
   (b) The second derivative f'' is positive on: ________________and negative on: ___________________.
   (c) List the points of inflection: $x =$
   (d) Does f have any relative extrema?
   (e) Does f have any absolute extrema?



Answer:
(a) down, up, down
(b) $(2,3)$, $(-\infty,2)\cup(3,\infty)$;
(c) $x=2,3$
(d) None
(e) Abs max at $x=3.5$, no abs min.

21. Suppose the position of a particle moving on a straight line is $s(t)=\sqrt{t}+4t^2$. Find the particle's acceleration as a function of time t.

    Answer: $a(t)=s''(t)=-\frac{1}{4}t^{-\frac{3}{2}}+8$

22. Let $s(t)=4e^t-8t^2+3$ be the position function of a particle moving in a straight line, where s is measured in feet and t is measured in seconds. Find its acceleration when $t=\ln 6$ seconds.

    Answer: 8 ft/sec^2

Related Rates (Section 12.5)

23. The radius of a circular puddle is growing at a rate of 15 cm/sec.
    (a) How fast is its area growing at the instant when the radius is 30 cm?
    (b) How fast is the area growing when the area is 81 square centimeters?

    Answer: (a) 2827 cm^2/sec; (b) 479 cm^2/sec

24. A rather flimsy spherical balloon is designed to pop at the instant its radius has reached 6 cm. Assuming the balloon is filled with helium at a rate of 13 cubic centimeters per second, calculate how fast the radius is growing at the instant it pops.

    Answer: 0.03 cm/sec

25. The average cost for the weekly manufacture of retro portable CD player is given by
    $$\bar{C}(x)=120{,}000x^{-1}+20+0.0004x \text{ dollars per player,}$$
    where x is the number of CD players manufactured that week. Weekly production is currently 4,000 players and is increasing at a rate of 400 players per week. What is happening to the average cost? Fill in the blank.

    The average cost is ____________increasing/decreasing at a rate of _____dollars per player per week.

    Answer: When $x=4000$, $\frac{d\bar{C}}{dt}=\frac{d\bar{C}}{dx}\cdot\frac{dx}{dt}=\$-2.84$ per player per week. So, the average cost is decreasing at a rate of 2.84 dollars per player per week.

Elasticity (Section 12.6)

26. Suppose the elasticity of demand is 3.2, when the price of a product is \$25. This means the demand is going up/down by ______% for 1% increase in the price. A small increase in price will result in a increase/decrease in the revenue.

    Answer: down, 3.2%, decrease.

27. Suppose the elasticity of demand is 0.65, when the price of a product is $\$500$. This means the demand will go up/down by ______% for 1% increase in the price. A small increase in price will cause the revenue to increase/decrease.

***Answer*:** down, 0.65%, increase.

28. The weekly sales of some backpacks is given by $q = 1080 - 18p$, where the q represents the quantity of backpacks sold at price p.
(a) Find the elasticity of demand at the price of \$20. Interpret your answer.
(b) Is the demand at the price \$20 elastic, inelastic, or unit elastic? Should the price be raised or lowered from \$20 to increase the revenue?
(c) What price will maximize the revenue?
(d) What is the maximum weekly revenue?

***Answer*:**

(a) $E(p) = -(-18) \cdot \frac{p}{1080-18p} = \frac{18p}{1080-18p}$, so $E(20) = \frac{18 \cdot 20}{1080-18 \cdot 20} = 0.5$.
This means the demand will drop by 0.5% for 1% increase from current price \$20.
(b) $0.5 < 1$, it is inelastic. The price should be raised to increase revenue.
(c) Solve for the price when $E(p) = 1$. Solving $\frac{18p}{1080-18p} = 1$ gives $p = \$30$.
(d) $R = pq = 30(1080 - 18 \cdot 30) = 16200$ dollars.

29. Suppose the demand function is $q = -2p^2 + 33p$, where q represents the quantity sold at price p.
(a) Find the price elasticity of demand $E(p)$.
(b) Find the elasticity when $p = \$15$. If the price increase by 1%, the demand will drop by how much? Should the price be lowered or raised from \$15 to increase the revenue?

***Answer*:**

(b) $E(p) = -(-4p + 33) \cdot \frac{p}{-2p^2+33p} = \frac{4p-33}{-2p+33}$
(c) $E(15) = \frac{4(15)-33}{-2(15)+33} = \frac{27}{3} = 9 > 1$. It is elastic. The demand will drop by 9% if the price increases by 1%.
The price should be lowered from \$15 to increase revenue.