MAT 210 Exam 1 Review Questions

Limits (sections 10.1, 10.3)

1. Calculate the limit: $\lim_{x\to 0}\frac{x-14}{x-7}$ Answer: 2

2. Calculate the limit: $\lim_{x\to +\infty} 2e^{-x}$ Answer: 0

3. Calculate the limit: $\lim_{x\to \infty}\frac{5-8\,x^2}{7-9x-4x^2}$ Answer: 2

4. Calculate the limit: $\lim_{x\to -\infty}\frac{4-8\,x^2}{2x+6}$ Answer: ∞

5. Calculate the limit: $\lim_{x\to \infty}\frac{3-2\,x^2}{2-4x-x^2+x^5}$ Answer: 0

6. The amount of drug (in milligrams) in the blood after an IV tube is inserted is given by $m(t) = 40 * 0.62^t$, where t is the number of hours after it was injected.
   Compute $\lim_{t\to +\infty} m(t)$, then interpret the result.
   Answer: $\lim_{t\to +\infty} m(t) = 0$. In the long term, the amount of drug in the blood will completely disappear.

7. The population of a colony of squirrels is given by $p(t) = \frac{1500}{3+2e^{-0.1t}}$, where t is the time in years since 1975.
   Compute $\lim_{t\to +\infty} p(t)$, then interpret the result.
   Answer: $\lim_{t\to +\infty} p(t) = 500$. The population of squirrels will approach 500 as the time increases (in the long run).

8. The following models approximate the popularity of Twitter and LinkedIn among social media sites from 2008 to 2013, as rated by statecounter.com:

   Twitter $W(t) = 0.33t^2 - 2t + 8.7$ percentage points

   LinkedIn $L(t) = 0.04t^2 - 0.26t + 0.67$ percentage points

   where t is the number of years since the start of 2008.

   a) Compute $\lim_{t\to +\infty} W(t)$, then interpret the result.
   Answer: $\lim_{t\to +\infty} W(t) = \infty$. In the long term, the popularity of Twitter among social media sites will increase without bound.

   b) Compute $\lim_{t\to +\infty} \frac{W(t)}{L(t)}$, then interpret the result.
   Answer: $\lim_{t\to +\infty} \frac{W(t)}{L(t)} = 8.25$. In the long term, the popularity of Twitter will increase by 8.25 more than LinkedIn.

   Note: A percentage can't rise beyond 100, so extrapolating the models to obtain long term predictions gives meaningless results.

Rates of Change (sections 10.4, 10.5)

1. Let $f(x) = x^3 + 2$. Find the average rate of change of f over the interval [1,4].
   Answer: 21

2. Calculate the average rate of change of the given function over the interval [2,6] and specify the unit of measurements.

x (days)	1	2	3	4	5	6	7	8	9
$f(x)$ (dollars)	6	20	-3	8	4.6	12	1.5	4.9	-9

   Answer: - 2 dollars/day

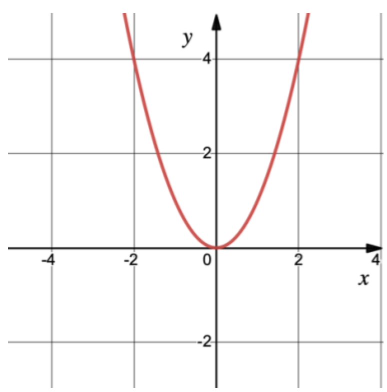
3. Suppose the following table shows U.S. daily oil imports from a certain country, for 1991−1999 (t = 1 represents the start of 1991). Use the data in the table to compute the average rate of change of $I(t)$ over the period 1991−1999 and interpret the meaning of the result.

***t* (year since 2000)**	1	2	3	4	5	6	7	8	9
***I*(*t*) (million barrels)**	5.6	2.5	3.8	9.4	8.2	3.7	9.6	7.2	8.1

   Answer: 0.3125 million barrels per year.

   U.S. daily oil imports from a certain country increased by an average rate of 0.3125 million barrels per year over the period 1991 to 1999.

4. Consider the following graph of f where f is defined over $(-\infty, \infty)$:

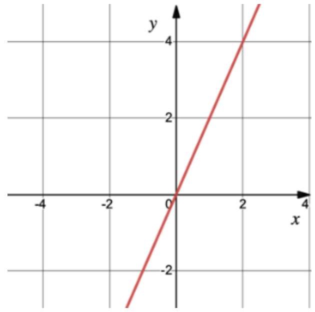


Interpret the sign of f':

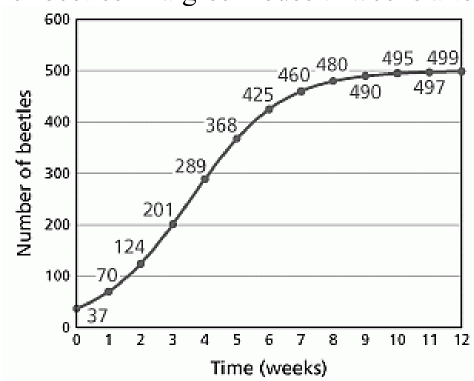
   Answer: The sign of f' is negative on the interval $(-\infty, 0)$, it is 0 at $x = 0$, then positive on the interval $(0, \infty)$

Graph f':

Answer: f' is always increasing



5. The graph below shows the population of beetles in a greenhouse t weeks after the season's flowers were planted.



a) Calculate the average rate of change over the interval [1,10].
Answer: 425/9 beetles per week

b) Circle **T** for True or **F** for False next to each statement.

T F a) During weeks [4,12] the instantaneous rate of change of the population is increasing

T F b) During weeks [4,12] the instantaneous rate of change of the population is decreasing

T F c) The average rate of change of the population on [4,12] is less than the instantaneous rate of change of the population at $t = 4$

T F d) The average rate of change of the population on [0,12] is greater than the instantaneous rate of change of the population at $t = 2$

T F e) The instantaneous rate of change of the population first increased then decreased

T F f) The instantaneous rate of change of the population at $t = 2$ is less than at $t = 11$

T F g) The instantaneous rate of change of the population at $t = 11$ is approximately 0

T F h) The instantaneous rate of change of the population at $t = 6$ is negative

T F i) In this graph, the slope of the tangent line at $t = 4$ is the greatest

6. Based on data from 1982 to 2017, the number of students taking the AP Calculus exam may be modeled by the function $S(t) = 157.8t^2 - 770.6t + 10268$, where t is the number of years since 1982. Interpret the meaning of $S(16) = 156{,}682$ and $S'(16) = 9{,}644$.

   A. In 1998 the number of students who took the AP Calculus exam is 156,682 and this number is increasing at a rate of 9,644 student per year.
   B. In 1998 the number of students who took the AP Calculus exam is 156,682 and this number is increasing at a rate of 9,644 student in every 16 years after 1982.
   C. In 1998 the number of students who took the AP Calculus exam is 156,682 and the test score is increasing at a rate of 9,644 points in every 16 years after 1982.
   D. In 1998 the number of students who took the AP Calculus exam is 156,682 and between 1982 and 2017 the number students who took the AP Calculus exam increased by an average of 9,644 students per year.
   E. None of these

Differentiation (Power Rule, Product Rule, Quotient Rule, Contant Multiple Rule) **(sections 11.1, 11.3)**

1. Find the derivative of each function using differentiation rules.
   a) $f(x) = 4x^2 + 2$ Answer: $f'(x) = 8x$

   b) $f(x) = 8 - 2x$ Answer: $f'(x) = -2$

   c) $f(x) = 2x^2 + 6x - 7$ Answer: $f'(x) = 4x + 6$

   d) $f(x) = \frac{8}{\sqrt{x}} - \frac{4}{x}$ Answer: $f'(x) = -4x^{-3/2} + 4x^{-2}$

   e) $g(x) = 3x^2 + 8x - \frac{2}{x^6} - 5x^{2.3} + 7.95$ Answer: $g'(x) = 6x + 8 + 12x^{-7} - 11.5x^{1.3}$

   f) $h(x) = 8x(3x^2 - 2x + 9)$ Answer: $h'(x) = 72x^2 - 32x + 72$

   g) $k(x) = \frac{3x+2}{4x}$ Answer: $k'(x) = -0.5x^{-2}$

   h) $m(x) = (-5x^2 + 7x^{-1})(x^{-2} - 7)$ Answer: $m'(x) = 70x - 21x^{-4} + 49x^{-2}$

   i) $v(x) = \frac{2x-1}{4x+8}$ Answer: $v'(x) = \frac{20}{(4x+8)^2}$

   j) $p(x) = \frac{x^2+1}{x^3-2x}$ Answer: $p'(x) = \frac{2x\cdot(x^3-2x)-(x^2+1)\cdot(3x^2-2)}{(x^3-2x)^2} = \frac{-x^4-5x^2+2}{(x^3-2x)^2}$

Applications to Derivatives and Rates of Change

Tangent Lines

1. Find the equation of the tangent line to the graph of $f(x) = 3x^2 - 4x + 6$ at $x = 2$.
   Answer: $y = 8x - 6$

2. Find the equation of the line tangent to the graph of $f(x) = x^2 - 2$ at $(3, 7)$.
   Answer: $y = 6x - 11$

3. Find the slope of the tangent line to the graph of $f(x) = 5x^3 + 2x + 4$ at $x = 1$
   Answer: $f'(1) = 17$

Marginal Analysis (section 11.2)

1. Assume that your monthly profit (in dollars) from selling homemade cookies is given by $P(x) = 8x - 2\sqrt{x}$, where *x* is the number of boxes of cookies you sell in a month.
   k) Determine the marginal profit function, *MP(x)*.

   Answer: $MP(x) = 8 - \frac{1}{\sqrt{x}}$

   l) Determine value of marginal profit if you are selling 25 boxes of cookies per month and interpret.

   Answer: 7.8 dollars per box. After 25 boxes of cookies have been sold, the total profit will increase by about 7.8 dollars per additional box sold, or the profit from selling the 26th box is about 7.8 dollars.

2. Find the marginal cost, the marginal revenue, and the marginal profit functions, where the cost and revenue functions, respectively, are $C(x) = 8x^2$, and $R(x) = 4x^3 + 2x + 10$.

   Answer: $MC(x) = 16x$
   $MR(x) = 12x^2 + 2$
   $MP(x) = 12x^2 - 16x + 2$

3. Assume that your monthly profit (in dollars) from selling books is given by $P(x) = 5x^2 + 6x - 2$, where *x* is the number of books you sell in a month. If you are currently selling $x = 50$ books per month, find your profit and your marginal profit.

   Answer: profit = 12798 dollars

   Marginal profit = 506 dollars per book

4. Your monthly cost (in dollars) from selling homemade candles is given by $C(x) = 150 + 0.1x + 0.002x^2$, where *x* is the number of candles you sell in a month. The revenue from selling x candles is $R(x) = 7x$.

a) Write a function $P(x)$ for your monthly *profit* of producing and selling x candles.
Answer: $P(x) = -150 + 6.9x - 0.002x^2$

b) Calculate $P(100)$. Include units.
Answer: 520 dollars

c) Write a function for your *marginal profit*.
Answer: $MP(x) = 6.9 - 0.004x$

d) Calculate your marginal profit if you produce and sell 100 candles. Include units and interpret your answer.

Answer: 6.5 dollars per candle. The profit from selling the 101^{st} candle is about 6.5 dollars. Or the total profit will increase by 6.5 dollars per candle sold, after 100 candles are sold.

Average Velocity and Instantaneous Velocity

1. Assume that the distance, s (in meters), traveled by a car moving in a straight line is given by the function $s(t) = t^2 - 3t + 5$, where t is measured in seconds.

a) Find the *average* velocity of the car during the time period from $t = 1$ to $t = 4$.

Answer: 2 m/s

b) Find the *instantaneous* velocity of the car at time $t = 3$ seconds.

Answer: 3 m/s