MAT 210 Limit Review for Exam 1

1. Calculate the limits of the following functions.

a. $\lim_{x\to 2} 2x^3 - 2x + 5\sqrt{x+2} = 22.$

b. $\lim_{x\to 3} \frac{x^2+x-6}{13x-26} = \frac{6}{13}.$

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2. Calculate the limits of the following rational functions.

a. $\lim_{x\to\infty} \frac{3x^3-3}{-13x^3-4x^2-2} = \lim_{x\to\infty} \frac{3-\frac{3}{x^3}}{-13-\frac{4}{x}-\frac{2}{x^3}} = \frac{3}{-13} = -\frac{3}{13}$, as $x \to \infty$ the read terms aproaches 0.

As $x \to \infty$, the numerator approaches ∞ and the denominator approaches $-\infty$. This limit expression has a $\frac{\infty}{-\infty}$ **type of indeterminate form (**Again this is not a number and not the answer.) Divide each term of the rational function by x^n, where n is the largest power of x in the expression. In this problem divide by x^3, where 3 is the largest power of x. Take the limit as $x \to \infty$, each $\frac{non-zero\ \#}{x^n}$ term approaches 0. Thus, for large x values (as $x \to \infty$ or as $x \to -\infty$), all terms of a rational function are negligible except the leading terms since the leading terms in both, in the numerator and denominator much larger compared to all the other terms. Therefore $\lim_{x\to\infty} \frac{3x^3-3}{-13x^3-4x^2-2} = \lim_{x\to\infty} \frac{3x^3}{-13x^3} = -\frac{3}{13}.$

In general, if the degree of the numerator of a rational function is the same as the degree of the denominator then the limit will be the ratio of the leading terms as $x \to \infty$ or as $x \to -\infty$.

b. $\lim_{x\to\infty} \frac{7x^2-5}{19x^3-3x-7} = \lim_{x\to\infty} \frac{7x^2}{19x^3} = \lim_{x\to\infty} \frac{7}{19x} = 0$.

As $x \to \infty$, the numerator approaches ∞ and the denominator approaches ∞. This limit expression has a $\frac{\infty}{\infty}$ **type of indeterminate form.** Again this is not a number and not the answer. In general, if the degree of the numerator of a rational function is smaller than the degree of the denominator then the limit is 0 as $x \to \infty$ or as $x \to -\infty$.

c. $\lim_{x\to\infty} \frac{-9x^5+5}{12x^3-3x-7} = \lim_{x\to\infty} \frac{-9x^5}{12x^3} = \lim_{x\to\infty} \frac{-9x^2}{12} = -\infty$.

As $x \to \infty$, the numerator approaches $-\infty$ and the denominator approaches ∞. This limit expression has a $\frac{-\infty}{\infty}$ **type of indeterminate form.** In general, if the degree of the numerator of a rational function is greater than the degree of the denominator then the limit does not exist. The limit is either ∞ or $-\infty$ depending on the sign of the leading coefficients.

All the limit expressions discussed above have $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$ indeterminate form, so there is no general rule to find the limit, each individual problem requires further analysis to determine the limit.

d. $\lim_{x\to\infty} \frac{-3}{11x^3} = 0$

As $x \to \infty$, the denominator approaches ∞. This limit expression has a $\frac{\textbf{non-zero \#}}{\infty}$ **type of determinate form,** thus the limit is 0. A non-zero number divided by a large number results a number a number that is close to 0. As the dividend increases without bound the value of the fraction is getting arbitrarily close to 0.

e. $\lim_{x\to\infty} \frac{-3x^3}{11} = -\infty$

As $x \to \infty$, the numerator approaches $-\infty$. This limit expression has a $\frac{-\infty}{\textbf{non-zero \#}}$ **type of determinate form,** thus the limit is $-\infty$.
