

**MEMORANDUM**

DATE: May 13, 2022

TO: Faculty and Students

FROM: Professor(s) Stephen Wirkus  
Chair/Co-Chairs of Sandra Cole  
Defense for the PhD in Applied Mathematics  
Committee Members Carl Gardner  
Erika Camacho  
John Fricks  
Nicolas Lanchier

**DEFENSE ANNOUNCEMENT**

Candidate: Sandra Cole  
Defense Date: 06/22/2022  
Defense Time: 10:00 AM  
Virtual Meeting Link: <https://asu.zoom.us/j/89124254044>  
Title: MODELING THE DYNAMICS OF HEROIN AND ILLICIT OPIOID USE DISORDER,  
TREATMENT, AND RECOVERY

Please share this information with colleagues and other students, especially those studying in similar fields. Faculty and students are encouraged to attend. The defending candidate will give a 40 minute talk, after which the committee members will ask questions. There may be time for questions from those in attendance. But, guests are primarily invited to attend as observers and will be excused when the committee begins its deliberations or if the committee wishes to question the candidate privately.

**ABSTRACT**  
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## ABSTRACT

A leading crisis in the United States is the opioid use disorder (OUD) epidemic. Opioid overdose deaths have been increasing, with over 100,000 deaths due to overdose from April 2020 to April 2021. These increases are due to over-prescribing opioids, switching from prescription opioids to heroin, and the infiltration of illicitly manufactured fentanyl. This dissertation presents two mathematical models to address illicit OUD, treatment, and recovery within an epidemiological framework. In the first model, individuals remain in the recovery class unless they relapse. Due to the limited availability of specialty treatment facilities for individuals with OUD, we incorporated a saturation treatment function. Our second model is an extension of the first, where we added a casual user class and its corresponding specialty treatment facilities class. Using U.S. population data from the CDC and SAMHSA (2002-2019), the data were scaled to a population of 200,000 to find parameter estimates. While our first model used the heroin-only dataset, the second model used both the heroin and all-illicit opioids datasets. We found backward bifurcation in the IOUD model with the heroin-only dataset for realistic parameter values. Additionally, we observed bi-stability in the IOUD model with a casual user with the heroin-only dataset. This result implies that it would be beneficial to increase the availability of treatment. An alarming result was discovered about the high overdose death rate: by 2038, the disease-free equilibrium would be the only stable equilibrium. This consequence is concerning because we do not want this to result from increasing opioid overdose deaths. Increasing police force efforts could help keep the deadly fentanyl off the streets. The IOUD model with a casual user class, PRCC results, and the comparison of parameters for both datasets, showed the importance of not overlooking the influence that casual users have in driving the all-illicit opioid epidemic. Casual users have a more significant influence, stay in the casual user class longer, and are not going to treatment as quickly as the users of the heroin epidemic. Another result was that the users of the all-illicit opioids were going to the recovered class by means other than specialty treatment. However, the relapse rates for those individuals were much more significant than in the heroin-only epidemic. Finally, the rate of an individual moving independently into treatment showed up consistently as significant for the first IOUD model. This result suggests decreasing stigmatization and promoting more readily available treatment options. The results above from analyzing these models may inform health and policy officials, leading to more effective treatment options and prevention efforts.