School of Mathematical and Statistical Sciences
Arizona State University

M E M O R A N D U M

DATE: May 3, 2022

TO: Faculty and Students

FROM: Professor(s) Marilyn Carlson
Chair/Co-Chairs of Franklin Yu
Defense for the PhD in Mathematics Education
Committee Members Kyeong Roh
Michelle Zandieh
Patrick Thompson
Roberto Soto

DEFENSE ANNOUNCEMENT

Candidate: Franklin Yu

Defense Date: 06/13/2022

Defense Time: 4:00 PM

Virtual Meeting Link: https://asu.zoom.us/j/89219285220 or ECA 385 (live)

Title: What is Instantaneous Rate of Change? An Investigation of Students' Conceptions and Learning of Instantaneous Rate of Change

Please share this information with colleagues and other students, especially those studying in similar fields. Faculty and students are encouraged to attend. The defending candidate will give a 40 minute talk, after which the committee members will ask questions. There may be time for questions from those in attendance. But, guests are primarily invited to attend as observers and will be excused when the committee begins its deliberations or if the committee wishes to question the candidate privately.

ABSTRACT
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Abstract

This dissertation reports on three studies about students' conceptions and learning of the idea of instantaneous rate of change. The first study investigated 25 students' conceptions of the idea of instantaneous rate of change. The second study proposes a hypothetical learning trajectory, based on the literature and results from the first study, for learning the idea of instantaneous rate of change. The third study investigated two students' thinking and learning in the context of a sequence of five exploratory teaching interviews.

The first paper reports on the results of conducting clinical interviews with 25 students. The results revealed the diverse conceptions that Calculus students have about the value of a derivative at a given input value. The results also suggest that students' interpretation of the value of a rate of change is related to their use of covariational reasoning when considering how two quantities' values vary together.

The second paper presents a conceptual analysis on the ways of thinking needed to develop a productive understanding of instantaneous rate of change. This conceptual analysis includes an ordered list of understandings and reasoning abilities that I hypothesize to be essential for understanding the idea of instantaneous rate of change. This paper also includes a sequence of tasks and questions I designed to support students in developing the ways of thinking and meanings described in my conceptual analysis.

The third paper reports on the results of five exploratory teaching interviews that leveraged my hypothetical learning trajectory from the second paper. The results of this teaching experiment indicate that developing a coherent understanding of rate of change using quantitative reasoning can foster advances in students' understanding of instantaneous rate of change as a constant rate of change over an arbitrarily small input interval of a function's domain.