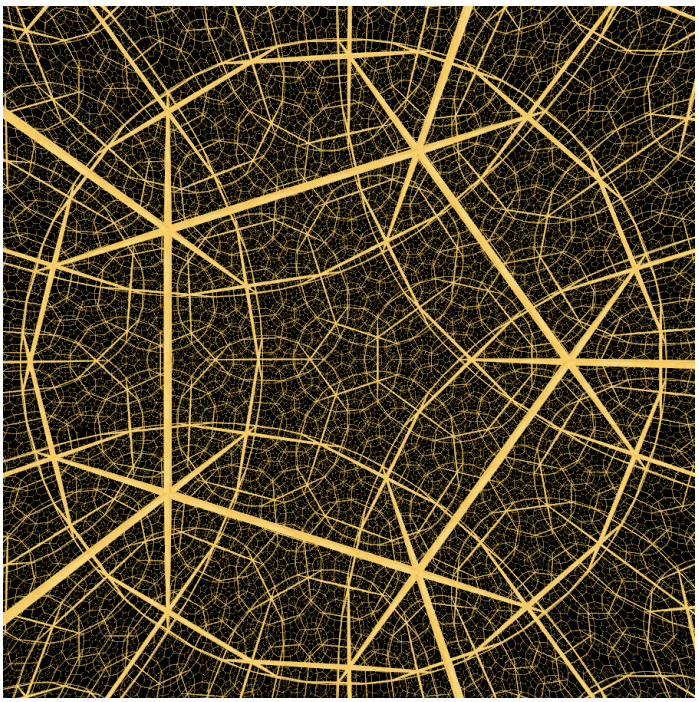
**Arizona State University**
**MAT 598, Fall 2022, MW 3-4:15**
**Instructor: Julien Paupert**

# Topics: Introduction to Hyperbolic Geometry



**Course description:** This course is an introduction to hyperbolic geometry at the graduate level. Topics include:

- **(I) Geometry of real and complex hyperbolic space**

  Models of hyperbolic space; isometries; totally geodesic subspaces; curvature; volume; configurations of triples and quadruples of points: angle invariants and cross-ratios; geometry of the boundary at infinity: conformal geometry and Heisenberg geometry.

- **(II) Discrete subgroups and lattices in** $\mathrm{Isom}(\mathrm{H}^n_{\mathbb{R}})$ **and** $\mathrm{Isom}(\mathrm{H}^n_{\mathbb{C}})$

  Basic definitions and properties; reflection groups and Coxeter groups; arithmetic groups: theory and examples; non-arithmetic constructions; structure theorems: Selberg lemma, Mostow rigidity, Margulis lemma.

**Prerequisites:** The prerequisites are the same as for admission to the Math PhD program: Advanced Calculus (MAT 371 or equivalent) and Linear Algebra (MAT 342 or equivalent). Some familiarity with groups and topology (connectedness, compactness, quotient spaces) is preferable but not required. References will be given to learn the minimal notions of these topics as needed.

**Course work:** There will be no homework or exams in this class, rather students will write a final project on the topic of their choice, related to any of the topics discussed in class.

**References for the class:**


- J. Paupert; Introduction to Hyperbolic Geometry. Lecture notes (Arizona State University, 2016). Available at:
  `https://math.la.asu.edu/ paupert/HyperbolicGeometryNotes.pdf`
- R. Benedetti and C. Petronio; Lectures on Hyperbolic Geometry. Universitext, Springer-Verlag (1992).
- W.M. Goldman; Complex Hyperbolic Geometry. Oxford Mathematical Monographs. Oxford University Press (1999).
- J.G. Ratcliffe; Foundations of Hyperbolic Manifolds, second edition. Graduate Texts in Mathematics Vol.149. Springer (2006).
- E.B. Vinberg (ed.); Geometry II. Encyclopaedia of Mathematical Sciences Vol. 29. Springer-Verlag (1991).
- D. Witte Morris; Introduction to Arithmetic Groups. Available at:
  `http://people.uleth.ca/~dave.morris/books/IntroArithGroups.html`


**Further reading:**


- A.F. Beardon; The Geometry of Discrete Groups, Graduate Texts in Mathematics Vol. 91, Springer (1983).
- S. Chen, L. Greenberg; *Hyperbolic spaces*, in Contributions to Analysis. Academic Press, New York (1974), 49–87.
- J.E. Humphreys; Reflection Groups and Coxeter Groups. Cambridge Studies in advanced mathematics vol. 29. Cambridge University Press (1990).
- M. Lackenby; Hyperbolic Manifolds. Lecture notes (Oxford 2000). Available at:
  `http://people.maths.ox.ac.uk/lackenby/`
- C. Maclachlan, A. Reid; The Arithmetic of Hyperbolic 3-Manifolds. Graduate Texts in Mathematics 219, Springer (2003).
- D. Mumford, C. Series, D. Wright (with cartoons by Larry Gonick); Indra's pearls : the vision of Felix Klein, Cambridge University Press (2002).
- W.P. Thurston; The Geometry and Topology of Three-Manifolds. Lecture notes (Princeton 1980). Available at: `http://library.msri.org/books/gt3m/`
- ...and references therein...