MAT 265 Exam Two Review Sections: 2.4-2.8, 3.1-3.3, 3.5, 3.7

Correct Answer is highlighted red - 39 questions

Section 2.4

1. Use the following table to answer question 1.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	3	2	1
2	3	4	3	4
3	2	1	4	2
4	1	2	1	3

If $H(x) = f(x) \cdot g(x)$, what is $H'(2)$?

24

2. Given $f(x) = 5\csc x$, find a) $f'(x)$ b) $f''(x)$.

$f'(x) = -5\csc x \cot x$; $f''(x) = -5(\csc x - 2\csc^3 x)$

3. The equation of motion for a particle is $s(t) = 5\cos t + 6\sin t$, $t \geq 0$, where s is measured in centimeters and t in seconds. Find the velocity function.

$s'(t) = -5\sin t + 6\cos t$

4. Find the derivative. $f(x) = x^{10}\cos x$

$f'(x) = 10x^9\cos x - x^{10}\sin x$

5. Find $f'(x)$ if $f(x) = 4x(\sin(x) + \cos(x))$

$f'(x) = 4(\sin(x) + \cos(x)) + 4x(\cos(x) - \sin(x))$

6. An object with weight P is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle *t* with the plane, then the magnitude of the force is $\mathrm{F} = \frac{cP}{c\,\sin t + \cos t}$, where *c* is a constant called the coefficient of friction. Let *P* = 30 lb and *c* = 0.5. When (in radians) is the rate of change of *F* with respect to *t* equal to zero?

arctan 0.5

Section 2.5

7. Find the first derivative of $y = \tan^4 x$.

$y' = 4\tan^3 x \sec^2 x$

8. Find the equation of the tangent line for $y = \cos^3(x)$ at $x = 0$.

$y = 1.$

Use the following table to answer question questions # 9 and # 10.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	1	3	2
2	2	2	1	3
3	3	1	3	1

9. If $h(x) = f(g(x))$, what is $h'(1)$?

6

10. If $H(x) = g(f(x))$, what is $H'(3)$?

3

11. Find the 28th derivative of $y = \cos(4x)$.

$y^{(28)} = 4^{28}\cos(4x)$

12. Find the derivative. $y = (\sec(x))^4 + \cos(x^5)$

$y' = 4(\sec(x))^4\tan(x) - 5x^4\sin(x^5)$

13. Suppose that $f(x) = \frac{3x}{(2-4x)^4}$. Find the equation of the tangent line of f at $x = 1$. Round each numerical value to 4 decimal places.

$y = -1.3125x + 1.5$

14. If 1000 dollars is invested at an annual interest rate r compounded monthly, the amount in the account at the end of 4 years is given by

$$A = 1000\left(1 + \frac{1}{12}r\right)^{48}$$

Find the rate of change of the amount A with respect to the rate r when $r = 4\%$

4677.204

Section 2.6

15. Find the slope of the tangent line to the curve $5xy^5 + 3xy = 24$ at $(3,1)$ exactly.

$-\frac{2}{21}$

16. For the equation given below, evaluate y' at the point $(2, 2)$ to six decimal places.

$(4x - y)^4 + 4y^3 = 1328.$

4.235294

17. Find the slope of the tangent line to the curve $5\sin x + 4\cos y - 4\sin x\cos y + x = 7\pi$ at $(7\pi, \frac{3\pi}{2})$.

1

Section 2.7

18. A street light is mounted on a 16 ft tall pole. A 6 ft woman walks away in a straight path from the pole at a speed of 4 ft/sec. How fast is the tip of the woman's shadow changing when she is 50 ft from the base of the pole?

6.4 ft/sec

19. If $x^2 + 3xy + y^5 = 39$, and $\frac{dx}{dt} = -2$ when $x = 1$ and $y = 2$, what is $\frac{dy}{dt}$ then?

16/83

20. The radius of a spherical balloon is increasing at a rate of 2 cm per min. How fast is the volume changing when the radius is 12 cm? Round your answer to six decimal places.

3619.114737

Section 2.8

21. Use a linear approximation to approximate $\sqrt{49.2}$. Write your answer to five decimal places.

7.01429

22. Let $y = 4\sqrt{x}$. To five decimal places: Find the change in y, Δy when $x = 4$ and $\Delta x = 0.2$.

0.19756

23. Let $y = 4\sqrt{x}$. To five decimal places: Find the differential dy when $x = 4$ and $dx = 0.2$.

0.2

24. Find linear approximation of the function $f(x) = \frac{1}{x}$ and use it to approximate $\frac{1}{1.04}$.

0.96

25. The radius of a circular disk is given as 24 cm with a maximal error in measurement of 0.2 cm.
    a) Use differentials to estimate the maximum error.
    b) What is the relative error?

Round each numerical value to 7 decimal places, except π. Leave π as π.

9.6π; 0.0166667

Section 3.1

26. Find the **exact** limit: $\lim_{x\to\infty} \frac{2\sqrt{11}(8)^x + 15{,}000}{7(8)^x - 9}$.

$\frac{2\sqrt{11}}{7}$

27. Find the **exact** limit: $\lim_{x\to-\infty} \frac{9}{5^x - 7}$.

$-\frac{9}{7}$

28. The number, N, of people who have heard a rumor spread by mass media at time, t, is given by

$$N(t)=a(1-e^{-kt}).$$

There are 200000 people in the population who hear the rumor eventually. 5 percent of them heard it on the first day. Find a and k, assuming t is measured in days.

a= 200000; $k=-\ln(1-5/100)$

Section 3.2

29. For the function $f(x) = 3x + 6x^{15}$, find the derivative of the inverse function of f at $c = -9$. In other words, find $(f^{-1})'(c)$ with $c = -9$.

$\frac{1}{93}$

30. Find the **exact** limit: $a)\ \lim_{x\to\infty} [\ln(5 + 3x) - \ln(5 + 2x)]$ $b)\ \lim_{x\to 0^+} [\ln 5 \sin x]$.
Round to six decimal places.

0.405465; $-\infty$

Section 3.3

31. Differentiate the function $f(x) = x^{6x}$.

$f'(x) = 6x^{6x}(\ln(x) + 1)$

32. Let $f(x) = -16\ln(\cos x)$. Find the second derivative of $f(x)$.

$f''(x) = 16sec^2x$

33. Let $f(x) = \ln[x^6(x + 5)^9(x^3 + 1)^{10}]$. Find $f'(x)$.

$f'(x) = \frac{6}{x} + \frac{9}{x + 5} + \frac{30x^2}{x^3 + 1}$

Section 3.5

34. Find $f'(x)$ where $f(x) = \arcsin^6(2x + 4)$.

$f'(x) = \frac{12\arcsin^5(2x+4)}{\sqrt{1-(2x+4)^2}}$

35. Let $f(x) = 2x^2 \tan^{-1}(8x^2)$. Find $f'(x)$.

$f'(x) = 4x\tan^{-1}(8x^2) + \frac{32x^3}{1+64x^4}$

Section 3.7

36. Use L'Hospital's Rule to evaluate the limit exactly: $\lim_{x \to 0^+} 4\sin(x)\ln(x)$

0

37. Use L'Hospital's Rule to evaluate the limit exactly: $\lim_{x \to 0} \frac{6^x - 8^x}{x}$.

$\ln\left(\frac{3}{4}\right)$

38. Use L'Hospital's Rule to evaluate the limit exactly: $\lim_{x \to \infty} (1 + \frac{8}{x})^{\frac{x}{10}}$

$e^{4/5}$

39. Use L'Hospital's Rule to evaluate the limit exactly: $\lim_{x \to \frac{\pi}{2}} (7\cos(-5x)\sec(-7x))$

-5