Section 2.4

1. Use the following table to answer question 1.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>g(x)</th>
<th>f'(x)</th>
<th>g'(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
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<tr>
<td>2</td>
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<td>3</td>
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<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

If \( H(x) = f(x) \cdot g(x) \), what is \( H'(2) \)?

24

2. Given \( f(x) = 5 \csc x \), find a) \( f'(x) \)  b) \( f''(x) \).

\( f'(x) = -5 \csc x \cot x \); \( f''(x) = -5(\csc x - 2 \csc^3 x) \)

3. The equation of motion for a particle is \( s(t) = 5 \cos t + 6 \sin t \), \( t \geq 0 \), where \( S \) is measured in centimeters and \( t \) in seconds. Find the velocity function.

\( s'(t) = -5 \sin t + 6 \cos t \)

4. Find the derivative. \( f(x) = x^{10} \cos x \)

\( f'(x) = 10x^9 \cos x - x^{10} \sin x \)

5. Find \( f'(x) \) if \( f(x) = 4x(\sin x) + \cos(x) \)

\( f'(x) = 4(\sin x) + \cos(x) + 4x(\cos(x) - \sin(x)) \)

6. An object with weight \( P \) is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle \( t \) with the plane, then the magnitude of the force is

\( F = \frac{cP}{\csc t + \cos t} \), where \( c \) is a constant called the coefficient of friction. Let \( P = 30 \text{ lb} \) and \( c = 0.5 \).

When (in radians) is the rate of change of \( F \) with respect to \( t \) equal to zero?

\( \arctan 0.5 \)
Section 2.5

7. Find the first derivative of \( y = \tan^4 x \).

\[ y' = 4 \tan^3 x \sec^2 x \]

8. Find the equation of the tangent line for \( y = \cos^3(x) \) at \( x = 0 \).

\[ y = 1. \]

Use the following table to answer question questions # 9 and # 10.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( f'(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2</td>
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<td>1</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

9. If \( h(x) = f(g(x)) \), what is \( h'(1) \)?

6

10. If \( H(x) = g(f(x)) \), what is \( H'(3) \)?

3

11. Find the 28th derivative of \( y = \cos(4x) \).

\[ y^{(28)} = 4^{28} \cos(4x) \]

12. Find the derivative. \( y = (\sec(x))^4 + \cos(x^5) \)

\[ y' = 4(\sec(x))^4 \tan(x) - 5x^4 \sin(x^5) \]

13. Suppose that \( f(x) = \frac{3x}{(2-4x)^4} \). Find the equation of the tangent line of \( f \) at \( x = 1 \).

Round each numerical value to 4 decimal places.

\[ y = -1.3125x + 1.5 \]

14. If 1000 dollars is invested at an annual interest rate \( r \) compounded monthly, the amount in the account at the end of 4 years is given by

\[ A = 1000 \left(1 + \frac{1}{12}r\right)^{48} \]

Find the rate of change of the amount \( A \) with respect to the rate \( r \) when \( r = 4\% \)

4677.204
Section 2.6

15. Find the slope of the tangent line to the curve \(5xy^5 + 3xy = 24\) at \((3,1)\) exactly.
\[
-\frac{2}{21}
\]

16. For the equation given below, evaluate \(y'\) at the point \((2,2)\) to six decimal places.
\[(4x - y)^4 + 4y^3 = 1328.
\]
\[4.235294\]

17. Find the slope of the tangent line to the curve \(5 \sin x + 4 \cos y - 4 \sin x \cos y + x = 7\pi\) at \((7\pi, \frac{3\pi}{2})\).
\[1\]

Section 2.7

18. A street light is mounted on a 16 ft tall pole. A 6 ft woman walks away in a straight path from the pole at a speed of 4 ft/sec. How fast is the tip of the woman’s shadow changing when she is 50 ft from the base of the pole?
\[6.4 \text{ ft/sec}\]

19. If \(x^2 + 3xy + y^5 = 39\), and \(\frac{dx}{dt} = -2\) when \(x = 1\) and \(y = 2\), what is \(\frac{dy}{dt}\) then?
\[16/83\]

20. The radius of a spherical balloon is increasing at a rate of 2 cm per min. How fast is the volume changing when the radius is 12 cm? Round your answer to six decimal places.
\[3619.114737\]
Section 2.8

21. Use a linear approximation to approximate \( \sqrt{49.2} \). Write your answer to five decimal places.
   7.01429

22. Let \( y = 4\sqrt{x} \). To five decimal places: Find the change in \( y \), \( \Delta y \) when \( x = 4 \) and \( \Delta x = 0.2 \).
   0.19756

23. Let \( y = 4\sqrt{x} \). To five decimal places: Find the differential \( dy \) when \( x = 4 \) and \( dx = 0.2 \).
   0.2

24. Find linear approximation of the function \( f(x) = \frac{1}{x} \) and use it to approximate \( \frac{1}{1.04} \).
   0.96

25. The radius of a circular disk is given as 24 cm with a maximal error in measurement of 0.2 cm.
   a) Use differentials to estimate the maximum error.
   b) What is the relative error?
   Round each numerical value to 7 decimal places, except \( \pi \). Leave \( \pi \) as \( \pi \).
   \( 9.6\pi; \ 0.0166667 \)

Section 3.1

26. Find the exact limit: \( \lim_{x \to \infty} \frac{2\sqrt{11}(8)^x + 15,000}{7(8)^x - 9} \).
   \( \frac{2\sqrt{11}}{7} \)

27. Find the exact limit: \( \lim_{x \to -\infty} \frac{9}{5^x - 7} \).
   \( -\frac{9}{7} \)

28. The number, \( N \), of people who have heard a rumor spread by mass media at time, \( t \), is given by \( N(t) = a(1 - e^{-kt}) \).
   There are 200000 people in the population who hear the rumor eventually. 5 percent of them heard it on the first day. Find \( a \) and \( k \), assuming \( t \) is measured in days.
   \( a = 200000; \ k = -\ln(1 - 5/100) \)
Section 3.2

29. For the function $f(x) = 3x + 6x^{15}$, find the derivative of the inverse function of $f$ at $c = -9$. In other words, find $(f^{-1})'(c)$ with $c = -9$.

\[
\frac{1}{93}
\]

30. Find the exact limit: 
   a) $\lim_{x \to \infty} [\ln(5 + 3x) - \ln(5 + 2x)]$
   b) $\lim_{x \to 0^+} [\ln 5 \sin x]$
   Round to six decimal places.

$0.405465; \ -\infty$

Section 3.3

31. Differentiate the function $f(x) = x^{6x}$.

$f'(x) = 6x^{6x} \ln(x) + 1$

32. Let $f(x) = -16 \ln(\cos x)$. Find the second derivative of $f(x)$.

$f''(x) = 16 \sec^2 x$

33. Let $f(x) = \ln[(x^6 + 5)^9(x^3 + 1)^{10}]$. Find $f'(x)$.

$f'(x) = \frac{6}{x} + \frac{9}{x + 5} + \frac{30x^2}{x^3 + 1}$

Section 3.5

34. Find $f'(x)$ where $f(x) = \arcsin^6(2x + 4)$.

$f'(x) = \frac{12 \arcsin^6(2x+4)}{\sqrt{1-(2x+4)^2}}$

35. Let $f(x) = 2x^2 \tan^{-1}(8x^2)$. Find $f'(x)$.

$f'(x) = 4x \tan^{-1}(8x^2) + \frac{32x^3}{1+64x^4}$
Section 3.7

36. Use L’Hospital’s Rule to evaluate the limit exactly: \( \lim_{x \to 0^+} 4 \sin(x) \ln(x) \)

0

37. Use L’Hospital’s Rule to evaluate the limit exactly: \( \lim_{x \to 0} \frac{6^x - 8^x}{x} \).

\( \ln \left( \frac{3}{4} \right) \)

38. Use L’Hospital’s Rule to evaluate the limit exactly: \( \lim_{x \to \infty} (1 + \frac{8}{x})^{10} \).

\( e^{4/5} \)

39. Use L’Hospital’s Rule to evaluate the limit exactly: \( \lim_{x \to \frac{\pi}{2}} (7 \cos(-5x) \sec(-7x)) \).

−5