MAT 598 (Topic class on Theory of elliptic partial differential equations)

Fall 2022

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TEXTBOOK: Partial Differential Equations, 2nd edition (Graduate Studies in Mathematics) by Lawrence C. Evans, **Chapters 5 and 6**.

COURSE DESCRIPTION: Sobolev spaces, approximation of Sobolev functions by smooth functions, Sobolev extension theorems, Sobolev trace theorems, Sobolev inequalities, Rellich-Kondrachov Compactness Theorem, second order uniformly elliptic equations, existence of a weak solution: the Lax-Milgram Theorem and Fredholm alternative, interior and boundary regularity of a solution, maximum principle and Harnack inequality.

PREREQUISITE: Degree- or nondegree-seeking graduate student.

COURSE OUTLINE: We will follow the following outline of topics. We might not be able to finish all the items listed below.

Some main topic on Sobolev Spaces

- Introducing the concept of weak partial derivatives motivated by integration by parts.
- Defining the Sobolev spaces $W^{k,p}$ whose members have up to order k weak derivatives lying in the L^p space, and proving that Sobolev spaces are Banach spaces.
- Approximating a Sobolev function by smooth functions using the method of mollifiers.
- *Extension Theorem*: Extending a Sobolev function in a bounded and sufficiently regular domain to a Sobolev function in the whole Euclidean space.
- *Trace Theorem*: Proving the existence of a bounded linear operator which assigns boundary values of a Sobolev function along the boundary of a sufficient regular open domain.
- Proving several embeddings of Sobolev spaces into others and some compactness results: *Gagliardo-Nirenberg-Sobolev Embedding Theorem*, *Morrey's Inequality*,*Rellich-Kondrachov Compactness Theorem*...
- Poincare inequality.
- Charaterization of H^{-1}, the dual space of the Sobolev space H_0^1.

Some main topics of the theory of elliptic PDEs

- Motivated by the approximation theorem of Sobolev functions by smooth functions, we will define what we mean by a weak solution of a BVP for a uniformely elliptic PDE written in divergence form.
- Analyzing the solvability of such a BVP using principles from linear functional analysis like The *Lax-Milgram theorem* and *Fredholm alternative*.
- Proving the interior and boundary H^2-regularity and higher regularity of an H_0^1-weak solution using some difference quotient techniques.
- *Weak and Strong Maximum Principles* which concern with the location of a maximum point (minimum point) of a subsolution (supersolution).