Recent Results on Sparse Graphs

Title: MAT 598 (81349) / MAT 494 (96929): Recent Results on Sparse Graphs
Time & Place: Fall 2022; Tu, Th 1:30–2:45 PM
Instructor: Kierstead
Description: We will study recent efforts to develop a theory for sparse graphs. Intuitively, a graph class is sparse if its members do not have "too" many edges, but this is not enough to be useful. Examples of sparse graph classes include classes with bounded maximum degree, classes defined by forbidding minors, especially the class of planar graphs, and classes with bounded tree-width. These classes have many interesting structural and computational properties. Our effort to identify and study sparse classes is motivated in part by the goal of finding more general classes with similar properties. On the other hand, results for general sparse classes can often be improved when applied to restricted classes. Nešetřil & Ossona de Mendez have introduced a surprisingly useful classification of sparsity using the notions of *classes with bounded expansion* which include the examples above, and even more generally, *nowhere dense classes*. It is remarkable that this classification has many, seemingly unrelated, equivalent formulations. The following theorem is an example of where their theory leads.

Theorem (Grohe, Kreutzer & Siebertz (2014))**.** *For every nowhere dense class $\mathcal{C}$ and $\varepsilon > 0$, every graph property definable in first-order logic can be decided in time $\mathcal{O}(n^{1+\varepsilon})$ on $\mathcal{C}$.*

The course will be run in a seminar format. Students will be asked to make regular presentations with the guidance and help of the instructor. It is hoped that this will promote intense in-class discussions.

References:

(1) Marcin Pilipczuk & Michał Pilipczuk, Course Notes: Sparsity.
https://www.mimuw.edu.pl/∼mp248287/sparsity2/
(2) J. Nešetřil and P. Ossona de Mendez, *Sparsity—Graphs, Structures, and Algorithms.*
Springer-Verlag, Berlin, Heidelberg, 2012.
(3) Many recent research papers

Prerequisites: Students should be familiar with basic graph theory, including proof techniques, roughly at the level of MAT 416/513 or higher. Detailed knowledge of specific theorems and proofs is not required, but some familiarity with planar graphs would help.

Audience: This dual undergraduate/graduate course will have combined lectures, but lower standards/expectations for the undergraduates (naturally there are no upper bounds on expectations). Students will be graded on class participation, with emphasis on their presentations.

The course should be useful for all graduate students interested in discrete mathematics, including computer science students interested in algorithms on graphs. Students will be introduced to research problems of current interest, as well as the process of reading papers and doing mathematical research. For undergraduates the course could form the basis for a senior honors thesis or NSF Graduate Fellowship application. For first-year graduate students it could form the basis for a summer block grant; more advanced students could use the course for general breadth, to help choose a research topic or to support their research.