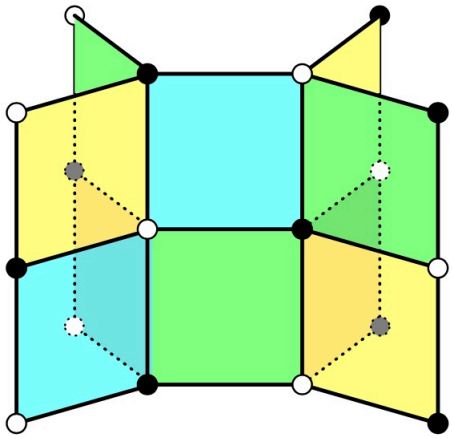
Arizona State University
MAT 598, Spring 2022

Geometric Group Theory with examples from Artin groups



Course Description: Groups are often investigated via representations as actions. For finite groups, group actions on finite sets (known as permutation representations) or on vector spaces (known as linear representations) have been extensively used with great success. Geometric group theory focuses on groups that can be analyzed using actions on topological spaces–particularly cell complexes and metric spaces–and oftentimes these groups have infinitely many elements. Motivated by the seminal works of Jim Cannon and Misha Gromov in the mid-1980s, group theorists have increasingly focused on the geometric structures that these cell complexes can carry and their connections to algebraic properties of a group. This course will offer a survey of topics in geometric group theory and consider applications of these techniques to Artin groups.

Artin groups are a class of (infinite) groups which are a far reaching generalization of the much better known braid groups. These groups have a simple algebraic description, but much is still unknown about them. However, we will consider some examples of Artin groups that lend themselves well to the geometric group theory material covered in this course. Time permitting, we will also acknowledge some gaps that remain in the understanding of Artin groups.

Topics:

- Artin/Coxeter groups, e.g. braid groups and symmetric groups
- Graphs, free groups, cell complexes, presentations
- Cayley graphs, metrics on groups, and geometric properties of groups
- Algorithms and decision problems
- Combinatorial curvature
- Bass-Serre Theory: Graph of groups, graph of spaces

Prerequisites: MAT 501 or familiarity with group theory and an understanding of fundamental groups and covering spaces will be a necessary prerequisite. We will be relying heavily on the material that can be found in Chapters 0 and 1 of Algebraic Topology by Allen Hatcher, freely available at https://pi.math.cornell.edu/ hatcher/AT/ATpage.html

Coursework: There will be no graded homework. However, suggested problems will be provided with opportunities to present solutions in lieu of a final paper at the end of the semester.