MAT 265 Exam One Review Sections: 1.3-1.6, 2.1-2.3

Section 1.3

1. **Numerically or algebraically** calculate the following limit **exactly**: $\lim_{x\to 0} \sin\left(\frac{200\pi}{x}\right)$.

2. **Numerically or algebraically** calculate the following limit **exactly**: $\lim_{x\to 1} \frac{5-5x}{1-\sqrt{x}}$.

3. Sketch the graph of the function $f(x) = \begin{cases} -x, & \text{if} \quad x \le -8 \\ 64 - x^2, & \text{if} \quad -8 < x < 8 \\ x+1, & \text{if} \quad x \ge 8 \end{cases}$

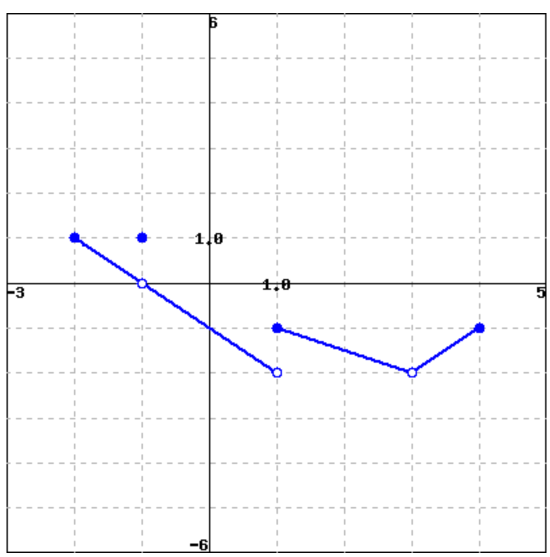
$\lim_{x\to 8^-} f(x) =$ ______, $\lim_{x\to 8^+} f(x) =$ ______, $\lim_{x\to 8} f(x) =$ ______

$\lim_{x\to -8^-} f(x) =$ ______, $\lim_{x\to -8^+} f(x) =$ ______, $\lim_{x\to -8} f(x) =$ ______

4. Find the following limits for the function whose graph is below:

$\lim_{x\to -1^-} f(x) =$ ______, $\lim_{x\to -1^+} f(x) =$ ______, $\lim_{x\to -1} f(x) =$ ______

$\lim_{x\to 1^-} f(x) =$ ______, $\lim_{x\to 1^+} f(x) =$ ______, $\lim_{x\to 1} f(x) =$ ______



5. Guess the value of limit (if it exists) by evaluating the function at the given numbers (correct to five decimal places): $-5.99, -5.999, -5.999, -6.01, -6.001, -6.001$.

$$\lim_{x\to -6} \frac{5x+30}{x^2+2x-24}$$

6. Guess the value of limit (if it exists) by evaluating the function at the given numbers (correct to five decimal places): $64.01, 64.001, 64.0001, 63.99, 63.999, 63.999$.

$$\lim_{y \to 64} \frac{64 - y}{8 - y^{0.5}}$$

Section 1.4

7. Evaluate the limit $\lim_{\theta \to \pi/2} 5\theta \sin \theta$

8 **Algebraically** calculate the **exact** limit $\lim_{h \to 0} \frac{\frac{8}{a+h} - \frac{8}{a}}{h}$.

9. **Algebraically** calculate the **exact** limit: $\lim_{h \to 0} \frac{3(a+h)^2 - 3a^2}{h}$.

10. **Algebraically** calculate the exact limit: $\lim_{x \to 10} \frac{x-10}{x^3-1000}$.

11. **Algebraically** calculate the following limit **exactly**: $\lim_{h \to 0} \frac{\sqrt{5(a+h)} - \sqrt{5a}}{h}$.

12. **Algebraically** calculate the following limit **exactly**:

$$\lim_{x \to 0} \frac{\sin(4x)}{9x}$$

13. **Algebraically** calculate the following limit **exactly**:

$$\lim_{x \to -9} \frac{x^2 - 81}{2x^2 + 22x + 36}$$

14. The figure shows a circular arc of length s and a chord of length d, both subtended by a central angle θ. Find $\lim_{\theta \to 0+} (s/d)$

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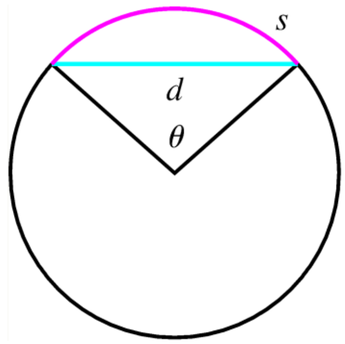

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Section 1.5

15. For the function $f(x) = \begin{cases} 4x - 4 & \text{if} \quad x < 8 \\ \frac{3}{x+9} & \text{if} \quad x \geq 8 \end{cases}$, answer the following questions.

a) $\lim_{x \to 8^-} f(x) =$ ______, b) $\lim_{x \to 8^+} f(x) =$ ______, c) $f(8) =$ ______

d) At $x = 8$, the function $f(x)$ has a jump discontinuity/removable discontinuity /infinite discontinuity or is continuous **(circle one).**

e) Explain your reasoning for your answer in part d.

16. Show there is a solution to the equation $\boldsymbol{x^3 - 0.5x^2 + 1.5 = 0}$ between $\boldsymbol{x = -2}$ and $\boldsymbol{x = 0}$. In the proof, be certain to state the theorem you used.

17. Let

$$f(x) = \frac{x - 8}{(x - 5)(x + 3)}$$

Use **interval notation** to indicate the largest set on which f is continuous.

18. Suppose a force exerted by an object on another mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \frac{2.8r}{R^6}, if \quad r < R \\ \frac{2.8R}{r^4}, if \quad r \geq R \end{cases}$$

Use the definition of continuity at a point to determine if F is continuous at $r=R$.

19. Redefine the function f to make it continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{9}{x} + \dfrac{-8x+36}{x(x-4)}; & x \neq 0, 4 \\ 4; & x = 0, 4 \end{cases}$$

20. For what value of the constant c is the function $f(x) = \begin{cases} cx^3 + x & \text{if } x \leq 2 \\ 5c - 3x & \text{if } x > 2 \end{cases}$ continuous on $(-\infty, \infty)$?

21. Let $f(x) = \frac{5}{x-8}$. Find the point of discontinuity of f and for each give the value of the point of discontinuity and evaluate the one-sided limits at that point.

Section 1.6

22. Calculate the limit **exactly**: $\lim_{x\to-\infty} f(x)$ where $\lim_{x\to-\infty} ((2x^2+2x)/(-5x^2+7500))$

23 Calculate the following limit **exactly**: $\lim_{x\to\infty} f(x)$ where $f(x) = \sqrt{144x^2+x} - 12x$.

24. Calculate the **exact** limit, $\lim_{x\to\infty} \frac{\sqrt{5x^2+25}}{9x+6}$.

25. Calculate the **exact** limit, $\lim_{x\to-\infty} \frac{\sqrt{2x^2+49}}{11x+1}$.

26. A population of organisms, in thousands, grows after *t* minutes according to the function:

$$C(t) = \frac{44t}{2+11t}$$

What does concentration approach as $t\to\infty$?

27. **Algebraically** find the following limit exactly: $\lim_{x\to\infty} \left(\sqrt{x^2+10} - \sqrt{x^2-10}\right)$.

Section 2.1

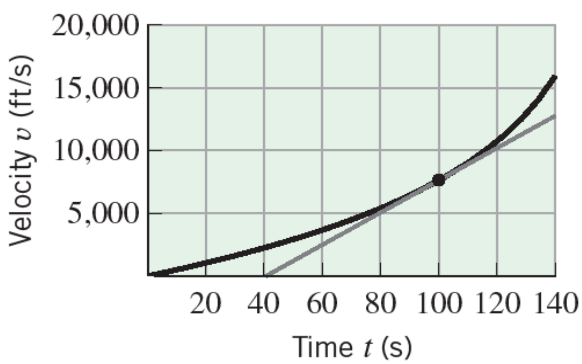
28. Let $f(x) = 6 - 2x^3$. Use the **limit definition of the derivative** to find the equation of the tangent line at $(2, -10)$. Write the answer in the form $y = mx + b$.

29. The accompanying figure shows the velocity versus time curve for a rocket in outer space where the only significant force on the rocket is from its engines. The mass $M(t)$ (in slugs) of the rocket at time t seconds satisfies the equation

$$M(t) = \frac{T}{dv/dt}$$

where *T* is the thrust (in lb) of the rocket's engines and *v* is the velocity (in *ft*/*s*) of the rocket. The thrust of the first stage of a rocket is *T*=10,000 lb.

Estimate the mass of the rocket at time *t*=100.



30. Suppose you deposit 1,000 dollars into a bank with 3% simple interest. The amount in the account after t years is given by $A(t)=1{,}000(1.03)^t$ (in dollars).

What is the average rate of change for the first year?
What is the average rate of change for the first six years, to two decimal places?

31. Let $g(x) = 2 - 3x^3$ Find $g'(1)$ and use it to find the equation of the tangent line to the curve $y = 2 - 3x^3$ at the point (1,-1). Write your answer in $y = mx + b$ form.

32. The limit $\lim_{h\to 0} \frac{\sqrt{64+h} - 8}{h}$ represents a derivative of some function $f(x)$ at some number a.

Find f and a.

33. The limit $\lim_{h\to 0}\frac{(8+h)^2-64}{h}$ represents a derivative of some function $f(x)$ at some number a.

Find f and a.

Section 2.2

34. Let $f(x)=\frac{5}{x}$.

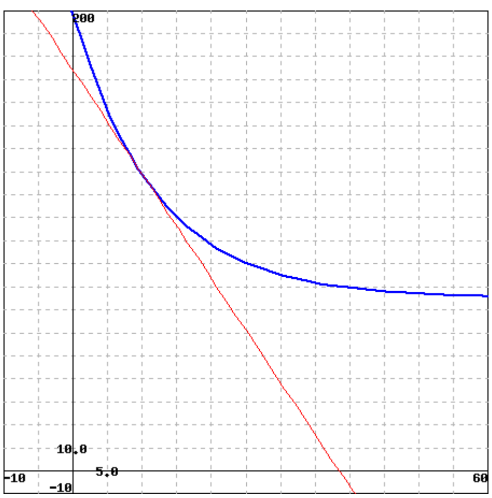
(A) Find and simplify $\frac{f(x+h)-f(x)}{h}$. Then use it and the **limit definition of the derivative** to find the derivative.

B) Find $f'(2)$.

35. According to *Newton's Law of Cooling*, the rate of change of an object's temperature is proportional to the difference between the temperature of the object and that of the surrounding medium. The accompanying figure shows the graph of the temperature T (in degrees Fahrenheit) versus time t (in minutes) for a cup of coffee, with initial temperature 200 degrees Fahrenheit, that is allowed to cool in a room with a constant temperature of 75 degrees Fahrenheit.

(a) Estimate T when t=10 minutes.

(b) Estimate dT/dt when t=10 minutes



Newton's Law of Cooling can be expressed as $dT/dt=k(T-T_0)$, where k is the constant of proportionality and T_0 is the temperature of the surrounding medium.

(c) Use the results of parts (a) and (b) to estimate the value of k.

36. Which of the following are true? List each letter in the ANSWER GRID. There may be more than one answer.

If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.

If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$.

If $f(x)$ is continuous at $x = a$, then then $f'(a)$ exists.

If $f'(2)$ exists, then $\lim_{x\to 2} f(x)$ exists.

If $f'(2)$ =1, then $\lim_{x\to 1} f(x)$ =1.

If $f(x) = (\sqrt{2})^{4.5}$, then $f'(x) = 4.5(\sqrt{2})^{3.5}$.

None of the choices

Section 2.3

37. If $f(x) = 7\sqrt{x}(x^3 - 8\sqrt{x} + 5)$, find $f'(x)$.

38. If $f(x) = \frac{2x^5 - 3x^4 - 7x^3}{x^4}$, find $f'(x)$.

39. At what point does the normal to $y = 3 - 2x + 2x^2$ at (1,3) intersect the parabola a second time?

40. Find the equation to the tangent line to the function $f(x) = 4\sin(x)$ at $(\frac{\pi}{6}, 2)$.

41. Let $y = (3 + 6x)^2$. Find the equation of A) the tangent line and B) the normal line at $(2, 225)$.

42. The position of a body moving along the *s*-axis is given by $s = \left(\frac{1}{3}\right)t^3 - 5t^2 + 25t + 8$, with s in meters and *t* in seconds. Find the body's acceleration a) after one second and b) each time the velocity is zero. If there are no answers, choose *NONE*.

43. If $f(x) = 100 + \frac{5}{x} + \frac{40}{x^2}$, find $f'(x)$.

44. If a ball is thrown vertically upward from a roof of a 32-foot high building with a velocity of 112 ft/sec its height, in feet, after t seconds is $s(t) = 32 + 112t - 16t^2$
(a) What is the maximum height, in feet, the ball reaches?
(b) What is the velocity, in feet per second, of the ball when it hits the ground (height 0)?

MAT 265 Exam One Review Answers Sections: 1.3-1.6, 2.1-2.3

1. does not exist

2. 10

3. 0, 9, DNE, 8, 0, DNE

4. 0, 0, 0, -2, -1, DNE

5. -0.5

6. 16

7. $\frac{5\pi}{2}$

8. $-\frac{8}{a^2}$

9. $6a$

10. $\frac{1}{300}$

11. $\frac{\sqrt{5}}{2\sqrt{a}}$

12. $\frac{4}{9}$

13. $9/7$

14. 1

15. A) 28 B) 3/17 C) 3/17 D) jump discontinuity; E) because $\lim_{x\to 8^-} f(x) \neq \lim_{x\to 8^+} f(x)$.

16.

By the Intermediate Value Theorem with $\boldsymbol{f(x) = x^3 - 0.5x^2 + 1.5}$, since $\boldsymbol{f(-2) < 0}$ and $\boldsymbol{f(0) > 0}$, there exists a real number $\boldsymbol{c}$ between $\boldsymbol{-2}$ and $\boldsymbol{0}$ such that $\boldsymbol{f(c) = 0}$.

17. $(-infinity, -3) \cup (-3,5) \cup (5, infinity)$

18. **No, unless $R^5 = R^3$. (That is $R = 1, -1$ or 0)**

19. $-\frac{1}{4}$

20. $c = -8/3$

21.

C =8

$\lim_{x \to 8^-} f(x) = -\infty$, $\lim_{x \to 8^+} f(x) = \infty$

22. $-\frac{2}{5}$

23. $\frac{1}{24}$

24. $\frac{\sqrt{5}}{9}$

25. $-\frac{\sqrt{2}}{11}$

26. 4,000

27. **Algebraically** find the following limit exactly: $\lim_{x \to \infty} (\sqrt{x^2 + 10} - \sqrt{x^2 - 10})$.

0

28.

Using $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ or $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$; $y = -24x + 38$

29. 80

30. 30, 32.34

31. $g'(1) = -9$ $y = -9x + 8$

32. $f(x) = \sqrt{x}$; $a = 64$

33. $f(x) = x^2$; $a = 8$

34.

A) $\frac{f(x+h)-f(x)}{h} = \frac{-5}{x(x+h)}$ so $f'(x) = -\frac{5}{x^2},$

B) $f'(2) = -1.25$

35.

(a) 128.964 (b) -4.53296 (c) -0.0839997

36. True:

If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.

If $f'(2)$ exists, then $\lim_{x \to 2} f(x)$ exists.

37. $f'(x) = 24.5x^{2.5} - 56 + 17.5x^{-0.5}$

38. $f'(x) = 2 + \frac{7}{x^2}$

39. $(-0.25, 3.625)$

40. $y = 4\frac{\sqrt{3}}{2}x + 2 - \frac{\sqrt{3}}{3}\pi$

41. $y = 180(x-2) + 225,\ \ y = -\frac{x-2}{180} + 225$

42. $-8, 0$

43. $f'(x) = -\frac{5}{x^2} - \frac{80}{x^3}$

44. (a) 228 ft (b) $-16\sqrt{57}$