APM 598 -- Fourier Analysis and Wavelets, Spring 2022

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Course Description

Fourier series and Wavelets are important mathematical building blocks for signal analysis and many other areas in science and engineering. Fourier series is the study of how a function (or signal) can be decomposed into a sum of sine and cosine waves of various frequencies. Wavelets are similar to sines and cosines in that they look like waves of various frequencies. However, they are different in that wavelets have localized support (unlike sine and cosine waves which are nonzero nearly everywhere). This localization feature of wavelets allows the user to filter or modify certain parts of the signal without affecting other parts.

This course will present an overview of Fourier and Wavelet Analysis along with some applications. The goal of this course is to present the general ideas behind the construction of Fourier series and Wavelets with a 50/50 mix of theory and computation/applications. The technical jargon of signal analysis and other fields of applications will be minimized. The primary applications to be covered include signal filtering (to emphasize or eliminate certain frequencies) and data compression (to reduce storage and speed up signal transmission).

No prior knowledge of Fourier series or wavelets will be assumed. The prerequisites are a two-semester calculus sequence, linear algebra (MAT 343) and being comfortable with epsilon-delta proofs in analysis. Some computer programming experience would be very helpful (especially with Matlab). This course should be accessible to undergraduate math/stat/dat majors who have had MAT 343 and 370.

Text

The recommended text for this course is *A First Course in Wavelets and Fourier Analysis, 2nd Edition*, by Boggess & Narcowich, 2009, Wiley. Lecture notes will also be provided. A copy of the text will be put on reserve in the library.

Other (optional) references include:

- *A First Course in Wavelets* by Hernandez, CRC Press, 1996 (in the library QA403.3.H47).

- *Wavelets and Other Orthogonal Systems with Applications*, by Walter, CRC Press, 1994 (in the library QA403.3.W34)
- *Wavelets, Mathematics and Applications*, edited by Benedetto and Frazier, CRC Press, 1993. See especially the article by Strichartz (pg 23-51) (in the library QA403.3.W4)
- *Introduction to Wavelets and Wavelet Transforms*, a Primer by Burns, Gopinath, Guo, Prentice Hall, 1998.
- *Wavelets, Algorithms and Applications*, by Meyer, SIAM Publications, 1993.
- *Fourier Series*, by Tolstov, Dover Press, 1962 (in library QA404.T573).

Grading

Grades will be determined by problem sets, one midterm exam and a final exam. Depending on the class size and schedule, exam reviews may involve students presenting solutions to practice exam questions.

The grade weights are as follows:
- Problem Sets – 30%
- Midterm – 30%
- Final Exam – 40%

The midterm and final exam will be in-class exams and you will be required to do your own work without help from others. You will be allowed a one-page sheet of notes during the exams but no other references (e.g. books or class notes).

You may consult with each other on homework problem sets, but only submit work, which is in your own words and be sure to cite any sources of help (e.g. reference or online materials or collaborators).

Tentative Schedule

Fourier series and inner products (4 weeks; chapter 1, parts of chapter 0)
Fourier transform (2 weeks; chapter 2)
Discrete Fourier analysis, including Fast Fourier Transform (1 week; chapter 3)
Haar wavelet (1.5 weeks; chapter 4)
Multiresolution analysis (2.5 weeks; chapter 5)
Daubechies wavelets (2 weeks; chapter 6)
Other wavelet topics (1 week; chapter 7)
(chapter numbers refer to the recommended text on Fourier/Wavelets)