Exam 1B MAT 272 - Platte February 12, 2015

Name PLATTE Student ID No. _____

SOLUTION

Directions:

- There are 5 multiple choice questions worth 7 points each, 4 true or false questions worth 4 points each, and 2 free response questions.
- You must show your work on all questions.
- You must give a clear and correct explanation or provide a counter-example for the true/false questions to receive credit.
- Partial credit is only available on the free response problems.
- Read all the questions carefully. No calculators with QWERTY keyboards or ones like TI-89 or TI-92 that do symbolic algebra may be used.
- Put the final answer to the space provided in an orderly fashion. Box your final answers. No partial credit will be given if more than one answer is given, or if it unclear which answer is meant to be your final answer.

Honor Statement

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and your instructor. Furthermore, you agree not to discuss this exam with anyone in any section of MAT 272 until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any exam proctor.

Signature:

Date:

1. The radius and center of the sphere $x^2 + y^2 + z^2 - 4x + 2y + 3 = 0$ are

   (a) $(2, -1, 0)$ and $\sqrt{2}$ $\quad (x-2)^2 + (y+1)^2 + z^2 + 3 - 4 - 1 = 0$

   (b) $(2, -1, 0)$ and 2 $\quad (x-2)^2 + (y+1)^2 + z^2 = 2$

   (c) $(2, -1, 0)$ and 3

   (d) $(-2, 1, 0)$ and 3

   (e) $(-4, 2, 0)$ and 3

   (f) Not enough information or none of the above

2. Which of the following is an equation for the straight line through the points (0,1,3) and (4,3,5).

   $\vec{v} = \langle 4, 2, 2 \rangle$

   (a) $\mathbf{r}(t) = \langle 0, 1, 3 \rangle + t \langle 4, 3, 5 \rangle$

   (b) $\mathbf{r}(t) = \langle 0, 1, 3 \rangle - t \langle 4, 3, 5 \rangle$

   (c) $\mathbf{r}(t) = \langle 4, 3, 5 \rangle + t \langle 8, 4, 4 \rangle$ — Parallel

   (d) $\mathbf{r}(t) = t \langle 4, 2, 2 \rangle$ — point on the line.

   (e) Not enough information or none of the above

3. The length of the curve $\mathbf{r}(t) = \langle 10\sqrt{2}t, e^{10t}, e^{-10t} \rangle$ on the interval $0 \le t \le 1$ is given by

   $r'(t) = \langle 10\sqrt{2}, 10e^{10t}, -10e^{-10t} \rangle$

   $r'(t) = 10 \langle \sqrt{2}, e^{10t}, -e^{-10t} \rangle$

   $|r'(t)| = 10\sqrt{2 + e^{20t} + e^{-20t}} = 10\sqrt{(e^{10t} + e^{-10t})^2}$

   $L = \int_0^1 10(e^{10t} + e^{-10t})\,dt = e^{10t} - e^{-10t}\Big|_0^1 = e^{10} - e^{-10}$

   (a) $e^5 - e^{-5}$

   (b) $e^{20} + e^{-20}$

   (c) $e^{10} + e^{-10}$

   (d) $e^{10} - e^{-10}$

   (e) Not enough information or none of the above

4. Let $\mathbf{v} = \mathbf{3i} + \mathbf{2k}$ and $\mathbf{w} = \mathbf{5i} - \mathbf{j}$. The vector projection of $\mathbf{w}$ onto $\mathbf{v}$ is

   $\text{Proj}_{\vec{v}} \vec{w} = \frac{\vec{w} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v} = \frac{(15)}{9+4} \vec{v}$

   (a) $\frac{10}{13} \langle 3, 2, 0 \rangle$

   (b) $\frac{15}{13} \langle 3, 0, 2 \rangle$

   (c) $\frac{10}{13} \langle 5, -1, 0 \rangle$

   (d) $\frac{10}{\sqrt{13}} \langle 3, 2, 0 \rangle$

   (e) Not enough information or none of the above

5. The angle between the vectors $\mathbf{a} = \langle 1, 1, 0 \rangle$ and $\mathbf{b} = \langle 0, 2, -2 \rangle$ is

   $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{2}{\sqrt{2}\sqrt{8}} = \frac{2}{2\sqrt{2}\sqrt{2}} = \frac{1}{2}$

   $\theta = \pi/3$

   (a) $\pi/3$

   (b) $\pi/6$

   (c) $\pi/4$

   (d) $\pi/2$

   (e) Not enough information or none of the above

1. True/False. Must CIRCLE the correct answer to receive credit.

4 pts each

(a) The magnitude of the cross product of a magnitude 4 vector and a magnitude 7 vector is always 28.

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta \Rightarrow |\vec{a}\times\vec{b}| \le |\vec{a}||\vec{b}|$ ($\sin\theta$ at most 1)

- true
- **false**

(b) The line $\mathbf{r}(t) = <3,-1,4> + t<6,-2,6>$ passes through the origin.

$<0,0,0> = <3+6t, -1-2t, 4+6t>$ ($t=-\frac{1}{2}$, $t=-\frac{1}{2}$, $t=-\frac{4}{6}=-\frac{2}{3}$)

- true
- **false**

(c) If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in $\mathbb{R}^3$, and $\mathbf{a}\cdot\mathbf{b} = \mathbf{a}\cdot\mathbf{c}$, then $\mathbf{b}$ and $\mathbf{c}$ differ by a vector that is parallel to $\mathbf{a}$.

$a\cdot b = a\cdot c \iff a\cdot(b-c) = 0$

$\iff a$ is $\perp$ to $(b-c)$

- true
- **false**

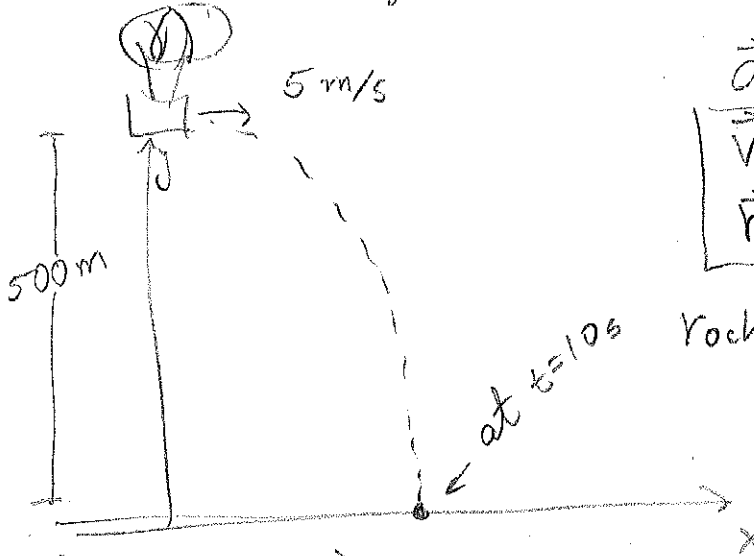
(d) Suppose a particle moves on a sphere centered at the origin in $\mathbb{R}^3$. That means that if $\mathbf{r}(t)$ is the vector function that describes the particle's trajectory, the $|\mathbf{r}(t)|$ is constant. Then the particle's velocity must be parallel to its position vector at all times.

- true
- **false**

example $\vec{r}(t) = <\cos t, \sin t, 0>$, $\vec{r}'(t) = <-\sin t, \cos t, 0>$ } not parallel

[24pts]

1. (10pts) A hot air balloon is flying at a constant altitude of 500 meters above the ground, and at a constant speed of 5 meters per second. Someone drops a rock from the balloon. Ignoring friction, and using $g = 10$ (meters/second^2), find the velocity and speed at which the rock hits the ground. (Model the situation by using an xy–coordinate system in which the x axis represents the ground, the balloon's location is $y = 500$ and its movement is in the positive x direction).



$\vec{a} = <0,-g> = <0,-10>$

$\vec{v} = <5, -10t>$

$\vec{r} = <5t, -5t^2+500>$

Rock hits the ground when $-5t^2+500 = 0$ or $t = 10$ s (at $t=10$s)

$\vec{r}_0 = <0, 500>$

$\vec{v}_0 = <5, 0>$

$\vec{v}(10) = <5, -100>$ velocity

$|\vec{v}(10)| = \sqrt{5+100^2}$ speed

≈ 100.125 m/s

2. (25 points) All following questions refer to a particle whose trajectory in time is given by $\mathbf{r}(t) =< 3\cos t, 2\sin t, \sqrt{5}\sin t >$, $0 \le t \le 2\pi$. The unit of length is meters and the unit of time is seconds.

(a) Find the speed at $t = \pi$.

4 pts

$\vec{r}'(t) = \langle -3\sin t, 2\cos t, \sqrt{5}\cos t \rangle$

$\vec{r}'(\pi) = \langle 0, -2, -\sqrt{5} \rangle$

$|\vec{r}'(\pi)| = \sqrt{4+5} = 3$

(b) Find the unit tangent vector at $t = \pi$.

6 pts

$\vec{T}(\pi) = \frac{1}{3}\langle 0, -2, -\sqrt{5} \rangle$

(c) What is the angle between the acceleration and velocity vectors at all times?

6 pts

$\vec{a}(t) = \langle -3\cos t, -2\sin t, -\sqrt{5}\sin t \rangle$

$\vec{r}' \cdot \vec{a} = 9\sin t\cos t - 4\cos t\sin t - 5\sin t\cos t = 0$

angle is $\pi/2$ at all times

(d) Find the tangent line to the trajectory at the point $(-3, 0, 0)$.

6 pts

$P(-3,0,0)$ corresponds to $t = \pi$

$\vec{v} = \langle 0, -2, -\sqrt{5} \rangle$

$\vec{r}(t) = \langle -3, 0, 0 \rangle + t\langle 0, -2, -\sqrt{5} \rangle$