Exam 3 MAT 272 - Platte

November 6, 2014

Name PLATTE

Student ID No. ________________

ANSWER KEY

Directions:

- There are 4 multiple choice questions worth 8 points each, 5 true false questions worth 4 points each, and 2 free response questions worth 24 points each.
- You must show your work on all questions, **including multiple choice.**
- Fort true/false questions, you must give a clear and correct explanation to justify your answer. If the anser is false, you may use a counter example.
- Partial credit is only available on the free response problems.
- Read all the questions carefully.
- You may not use your calculators to graph functions or integrate integrals.
- Put the final answer to the space provided in an orderly fashion. Box your final answers. No partial credit will be given if more than one answer is given, or if it unclear which answer is meant to be your final answer.

Honor Statement

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and your instructor. Furthermore, you agree not to discuss this exam with anyone in any section of MAT 272 until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any exam proctor.

Signature:

Multiple Choice

1. The volume of the region bounded by the plane $z = \sqrt{29}$ and the hyperboloid $z = \sqrt{4 + x^2 + y^2}$ can be expressed using a triple integral in cylindrical coordinates as

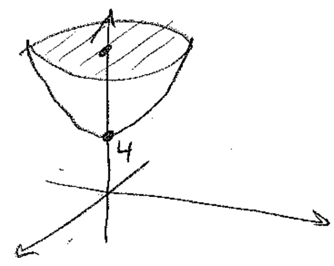
   (a) $\int_0^{2\pi} \int_0^2 \int_{\sqrt{4+r^2}}^{\sqrt{29}} r dz dr d\theta$

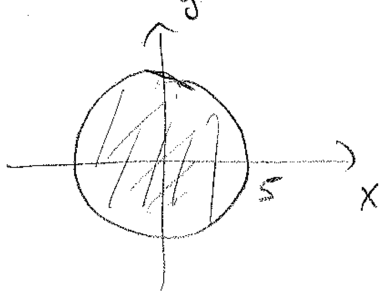
   (b) $\int_0^{2\pi} \int_0^5 \int_{\sqrt{4+r^2}}^{\sqrt{29}} r dz dr d\theta$

   (c) $\int_0^{2\pi} \int_0^2 \int_2^{\sqrt{29}} r dz dr d\theta$

   (d) $\int_0^{2\pi} \int_0^2 \int_0^{\sqrt{4+r^2}} r dz dr d\theta$

   (e) Not enough information or none of the above





$29 = 4 + x^2 + y^2$

$25 = x^2 + y^2$

2. The center of mass of the upper half unit sphere in $\mathbb{R}^3$, given by $\{(x, y, z) : x^2 + y^2 + z^2 \leq 1, z \geq 0\}$, having constant density C, is

   (a) $(0, 0, 0)$

   (b) $(0, 0, \frac{1}{2})$

   (c) $(0, 0, \frac{3}{8})$

   (d) $(0, 0, \frac{1}{4})$

   (e) Not enough information or none of the above

assume $\rho = 1$

mass $=$ Volume $= \frac{4\pi}{3} \frac{1}{2}$

$= \frac{2\pi}{3}$

$$m_{xy} = \iiint z \, dv$$

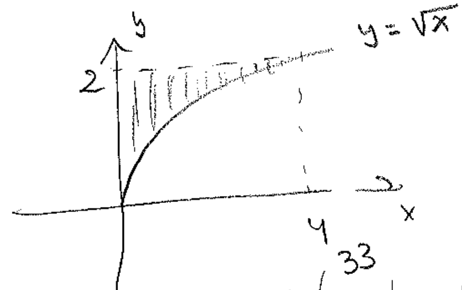
$$= \int_0^{2\pi} \int_0^{\pi/2} \underbrace{\int_0^1 (\rho \cos\varphi) \rho^2 \sin\varphi \, d\rho}_{\frac{\rho^4}{4} \cos\varphi \sin\varphi \Big|_0^1 = \frac{1}{4} \cos\varphi \sin\varphi} d\varphi \, d\theta$$

$$\int_0^{\pi/2} \cos\varphi \sin\varphi \, d\varphi = \frac{\sin^2\varphi}{2} \Big|_0^{\pi/2} = \frac{1}{2}$$

hence $m_{xy} = 2\pi \times \frac{1}{2} \times \frac{1}{4} = \frac{\pi}{4}$ and $\bar{z} = \frac{\pi}{4} \times \frac{3}{2\pi} = \frac{3}{8}$

3. The solution to $\int_0^4 \int_{\sqrt{x}}^2 (\frac{x}{y^5+1}) dy dx$ is (hint: reverse the order of integration):

(a) $\frac{\ln 33}{10}$
(b) 2
(c) $\frac{16}{5}$
(d) $\frac{\ln 10}{2}$
(e) Not enough information or none of the above

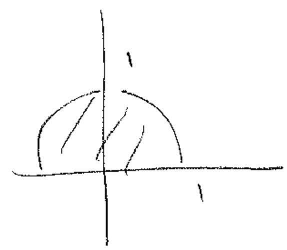


$$\int_0^2 \underbrace{\int_0^{y^2} \left(\frac{x}{y^5+1}\right) dx}_{\frac{1}{2}\frac{x^2}{(y^5+1)}\Big|_0^{y^2} = \frac{1}{2}\frac{y^4}{y^5+1}} dy = \frac{1}{2}\int_0^2 \frac{y^4}{y^5+1} dy = \frac{1}{10}\int_1^{33} \frac{1}{u} du = \frac{\ln 33}{10}$$

$u = y^5 + 1$
$du = 5y^4 dy$

4. The solution to $\int \int_R exp(x^2 + y^2) dy dx$, where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$ is

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{2}(e - 1)$
(c) 1
(d) $\frac{e}{3}$
(e) Not enough information or none of the above



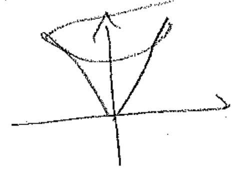
$$\int_0^{\pi} \underbrace{\int_0^1 e^{r^2} r\, dr}_{\frac{1}{2}e^{r^2}\Big|_0^1 = \frac{1}{2}(e-1)} d\theta = \frac{\pi}{2}(e-1)$$

True False

1. The sets $\{(r, \theta, z) : r = z\}$ and $\{(\rho, \phi, \theta) : \phi = \frac{\pi}{4}\}$ are the same.

True

both are cones



2. Changing the order of integration of $\int_0^2 \int_1^y f(x,y)dxdy$ gives $\int_1^y \int_0^2 f(x,y)dydx$.

False

3. Let R be the circle of radius one centered at $(0,0)$. Then $\int\int_R (x^2+y^2)dA = \int_0^{2\pi}\int_0^1 r^2 dr d\theta$.

False

$\int_0^{2\pi}\int_0^1 r^2 \, r dr d\theta$

4. In cylindrical coordinates, $z^2 = 9 - r^2$ is a sphere.

True

$z^2 = 9 - (y^2 + x^2)$

$x^2 + y^2 + z^2 = 9$

5. Assuming g is integrable and $a, b, c,$ and d are constants,

$$\int_c^d \int_a^b g(x,y)dxdy = \left(\int_a^b g(x,y)dx\right)\left(\int_c^d g(x,y)dy\right).$$

False

Free Response

1. (24 points) Show that the volume of a sphere with radius R is given by $V = \frac{4}{3}\pi R^3$ by setting up and evaluating a triple integral that describes that volume. You will not get partial credit if you do not use a triple integral to demonstrate this formula.

Spherical Coordinates

$$\int_0^{2\pi} \int_0^{\pi} \underbrace{\int_0^{R} \rho^2 \sin\varphi \, d\rho}_{\frac{1}{3}\rho^3 \sin\varphi \Big|_0^R = \frac{1}{3}R^3 \sin\varphi} \, d\varphi \, d\theta$$

$$\frac{1}{3}R^3 \int_0^{2\pi} \underbrace{\int_0^{\pi} \sin\varphi \, d\varphi}_{-\cos\varphi \Big|_0^{\pi} = 2} \, d\theta = \frac{2}{3}R^3 \int_0^{2\pi} d\theta = \frac{4\pi R^3}{3}$$

or

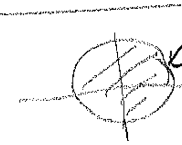
$z^2 + r^2 = R^2 \Rightarrow z = \pm\sqrt{R^2 - r^2}$

Cylindrical coordinates

$$\int_0^{2\pi} \int_0^{R} \int_{-\sqrt{R^2 - r^2}}^{\sqrt{R^2 - r^2}} r \, dz \, dr \, d\theta$$

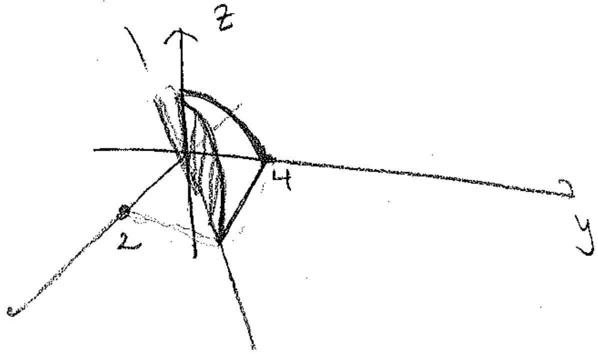
or

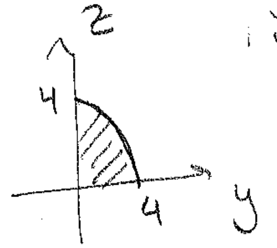
Cartesian coordinates



$$\int_{-R}^{R} \int_{-\sqrt{R^2 - x^2}}^{\sqrt{R^2 - x^2}} \int_{-\sqrt{R^2 - x^2 - y^2}}^{\sqrt{R^2 - x^2 - y^2}} dz \, dy \, dx$$

2. (24 points) Evaluate the triple integral $\iiint_E z\,dV$ where E is the solid bounded by the cylinder $y^2 + z^2 = 16$ and the planes $x = 0$, $y = 2x$ and $z = 0$ in the first octant.





$y = r\cos\theta$
$z = r\sin\theta$

Use polar coordinates in yz plane

$$\int_0^{\pi/2}\int_0^4\int_0^{\frac{1}{2}r\cos\theta} r\sin\theta \, r \, dx\,dr\,d\theta$$

$$= \int_0^{\pi/2}\int_0^4 \frac{1}{2}r^3\cos\theta\sin\theta\,dr\,d\theta$$

$$= \int_0^{\pi/2}\frac{1}{8}4^4\cos\theta\sin\theta\,d\theta$$

$$= 32\,\frac{\sin^2\theta}{2}\Big|_0^{\pi/2} = \boxed{16}$$

or

$$\int_0^2\int_{2x}^4\int_0^{\sqrt{16-y^2}} z\,dz\,dy\,dx$$

or

$$\int_0^4\int_0^{y/2}\int_0^{\sqrt{16-y^2}} z\,dz\,dx\,dy$$