Exam 2 MAT 272 - Platte October 16, 2014

Name PLATTE Student ID No. ____________

ANSWER KEY

Directions:

- There are 4 multiple choice questions worth 8 points each, 5 true false questions worth 4 points each, and 2 free response questions worth 24 points each.
- You must show your work on all questions, **including multiple choice.**
- Fort true/false questions, you must give a clear and correct explanation to justify your answer. If the anser is false, you may use a counter example.
- Partial credit is only available on the free response problems.
- Read all the questions carefully. No calculators with QWERTY keyboards or ones like TI-89 or TI-92 that do symbolic algebra may be used. **No graphing calculators are allowed.**
- Put the final answer to the space provided in an orderly fashion. Box your final answers. No partial credit will be given if more than one answer is given, or if it unclear which answer is meant to be your final answer.

Honor Statement

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and your instructor. Furthermore, you agree not to discuss this exam with anyone in any section of MAT 272 until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any exam proctor.

Signature:

Multiple Choice

1. The surface $4x^2 - y^2 + 2z^2 + 4 = 0$ is a(n)

   (a) elliptic paraboloid
   (b) hyperbolic paraboloid
   (c) ellipse
   (d) hyperboloid of two sheet
   (e) Not enough information or none of the above

2. Does the function $f(x,y) = \frac{x}{\sqrt{x^2+y^2}}$ have a limit as $(x,y) \to (0,0)$?

   (a) Yes, the limit is 1.
   (b) Yes, the limit is $\frac{1}{\sqrt{2}}$.
   (c) Yes, the limit exists but is not 1 or $\frac{1}{\sqrt{2}}$.
   (d) No, the limit does not exist because the domain of $f(x,y)$ includes only values such that $(x,y) \neq (0,0)$.
   (e) No, the limit does not exist because it has different values as 0 is approached from different paths to $(0,0)$.

Let $y=0 \Rightarrow \lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{x}{\sqrt{x^2}} = \lim_{x\to 0} \frac{x}{|x|}$ D.N.E

3. The rate of change of $f(x,y) = xe^y$ at the point $P(2,0)$ in the direction from P to $Q(\frac{1}{2}, 2)$ is

   (a) $\sqrt{5}$
   (b) $-\frac{1}{5}e^2$
   (c) 1
   (d) 2
   (e) Not enough information or none of the above

$\vec{v} = \vec{PQ} = \langle \frac{1}{2} - 2, 2-0 \rangle = \langle -3/2, 2 \rangle$, $|\vec{v}| = \sqrt{9/4 + 4} = \frac{5}{2}$

$\boxed{\vec{u} = \frac{2}{5}\langle -3/2, 2 \rangle}$ $\nabla f = \langle e^y, xe^y \rangle$

$\boxed{\nabla f(2,0) = \langle 1, 2 \rangle} \Rightarrow \boxed{D_{\vec{u}} f = \frac{-3}{5} + \frac{8}{5} = 1}$

4. If $z = f(x,y)$ where $x = g(t)$, $y = h(t)$, $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$, h'(3) = -4, $f_x(2,7) = 6$, and $f_y(2,7) = -8$ then $\frac{dz}{dt}$ when $t = 3$ is

   (a) 62
   (b) -44
   (c) -18
   (d) -2
   (e) Not enough information or none of the above

$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = f_x g' + f_y h'$

$= 6 \times 5 - 8 \times (-4)$

$= 30 + 32 = \boxed{62}$

True False

1. The planes tangent to the cylinder $x^2 + z^2 = 1$ in $\mathbb{R}^3$ all have the form $ax + bz + c = 0$.

   - true
   - false

2. Suppose $w = \frac{xy}{z}$ for $x > 0$, $y > 0$ and $z > 0$. A decrease in z with x and y fixed results in an increase in w.

   $$\frac{\partial w}{\partial z} = -\frac{xy}{z^2} < 0$$ w is decreasing with z.

   - true
   - false

3. If the limits $\lim_{(x,0)\to(0,0)} f(x,0) = L$ and $\lim_{(0,y)\to(0,0)} f(0,y) = L$, then $lim_{(x,y)\to(0,0)} f(x,y) = L$.

   - true
   - false

4. If plane Q is orthogonal to plane R, and plane R is orthogonal to plane S, then plane Q is orthogonal to plane S.

   - true
   - false

5. All level curves of the surface $z = 2x - 3y$ are lines.

   plane

   - true
   - false

1. Let $f(x, y) = y \ln x$.

   (a) (8 points) Find the direction of maximum increase of f at the point $(1, 4, 0)$. What is the rate of change in that direction?

   $$\nabla f = \left\langle \frac{y}{x}, \ln x \right\rangle$$

   $$\nabla f(1,4) = \langle 4, 0 \rangle$$ direction of max increase

(b) (8 points) Find the equation of the tangent plane to f at the point $(1, 4, 0)$.

$$z = 4(x-1) + 0(y-4) + 0$$

$$\boxed{z = 4x - 4}$$

(c) (8 points) Use the linearization of f at $(1, 4, 0)$ approximate $f(1.1, 3.9)$.

$$\mathcal{L}(x,y) = 4x - 4$$

$$\mathcal{L}(1.1, 3.9) = 4.4 - 4 = .4 \;/\!/$$

2. (24 points) Given the function $f(x, y) = x^2y - y^7 - x^2 + 6y$, find all critical points of f and use the second derivative test to identify them as maximum, minimum or saddle points.

$$\frac{\partial f}{\partial x} = 2xy - 2x = 0 \Rightarrow 2x(y-1) = 0 \Rightarrow \boxed{x=0} \text{ or } \boxed{y=1}$$

$$\frac{\partial f}{\partial y} = x^2 - 7y^6 + 6 = 0 \qquad \text{if } x = 0 \Rightarrow -7y^6 + 6 = 0$$

$$y = \pm\left(\frac{6}{7}\right)^{1/6}$$

$$\text{if } y = 1 \Rightarrow x^2 - 1 = 0$$

$$\Rightarrow \boxed{x = \pm 1}$$

Critical points

$\left(0, \pm\left(\frac{6}{7}\right)^{1/6}\right)$ and $(\pm 1, 1)$

Second derivative test

$$f_{xx} = 2y - 2$$

$$f_{yy} = -42y^5$$

$$f_{xy} = 2x$$

$$D(x,y) = f_{xx}f_{yy} - f_{xy}^2$$

$$D(\pm 1, 1) = (0)(-42) - 2^2 < 0$$

$\Rightarrow (\pm 1, 1)$ are saddle points

$$f_{xx}\left(0, \left(\tfrac{6}{7}\right)^{1/6}\right) < 0 \qquad f_{xy}\left(0, \left(\tfrac{6}{7}\right)^{1/6}\right) = 0$$

$$f_{yy}\left(0, \left(\tfrac{6}{7}\right)^{1/6}\right) < 0$$

$\Rightarrow \left(0, \left(\frac{6}{7}\right)^{1/6}\right)$ is a local max.

$$\left.\begin{array}{l} f_{xx}\left(0, -\left(\tfrac{6}{7}\right)^{1/6}\right) < 0 \\ f_{yy}\left(0, -\left(\tfrac{6}{7}\right)^{1/6}\right) > 0 \end{array}\right\} \Rightarrow \left(0, \left(\tfrac{6}{7}\right)^{1/6}\right) \text{ is a saddle point}$$