Sequences

1. Give the first six terms of the following sequences:

(a) $s_n = \frac{n(n+1)}{2}$

(b) $s_n = \frac{n + (-1)^n}{n}$

2. Give a general term for the following sequences:

(a) 1, 2, 4, 8, 16, 32, …

(b) $\frac{7}{2}, \frac{7}{5}, \frac{7}{8}, \frac{7}{11}, \frac{1}{2}, \frac{7}{17}, \ldots$

3. Do the following sequences converge or diverge? If a sequence converges, find its limit.

(a) $s_n = (0.8)^n$

(b) $s_n = \frac{1 - e^{-n}}{1 + e^{-n}}$

(c) $s_n = 1 + (-1)^n$

A sequence s_n is called ***monotone*** **i**f it is either increasing, that is $s_n < s_{n+1}$ for all n, or decreasing, that is $s_n > s_{n+1}$ for all n.

Theorem: Convergence of a Monotone, Bounded Sequence

If a sequence s_n is bounded and monotone, it converges.

Let $s_n = (1 + 1/n)^n$
What is the limit of s_n as n increases without bound?

4. Do the sequences in Problems below converge or diverge? If a sequence converges, find its limit.

a. 2^n

b. $(0.2)^n$

c. $3 + e^{-2n}$

d. $(-0.3)^n$

e. $\frac{n}{10} + \frac{10}{n}$

f. $\frac{2^n}{3^n}$

g. $\frac{2n+1}{n}$

h. $\frac{(-1)^n}{n}$

i. $\frac{2^n}{n^3}$

j. $\frac{\sin n}{n}$

k. The Fibonacci sequence first studied by the thirteenth century Italian mathematician Leonardo di Pisa, also known as Fibonacci, is defined recursively by

$$F_n = F_{n-1} + F_{n-2} \text{ for } n > 2 \text{ and } F_1 = 1, F_2 = 1.$$

The Fibonacci sequence occurs in many branches of mathematics and can be found in patterns of plant growth (phyllotaxis). Find the first 12 terms.

The following sequences are ordered according to increasing growth rates as *n* approaches infinity and the rankings do not change if a sequence is multiplied by a positive constant:

$\{\ln^q n\} \ll \{n^p\} \ll \{n^p \ln^r n\} \ll \{n^{p+s}\} \ll \{b^n\} \ll \{n!\} \ll \{n^n\}$

$p, q, r, s, b > 1$ are positive real numbers

Compare the growth rates of the following sequences:

1. $\{\ln n\} \,\&\, \{n^{1.1}\}$
2. $\{n^{1{,}000{,}000}\} \,\&\, \{e^n\}$
3. $\{\ln n^{10}\} \,\&\, \{0.00001n\}$
4. $\{n!\} \,\&\, \{10^n\}$

Infinite Series

Geometric Series

A person with an ear infection is told to take antibiotic tablets regularly for several days. Since the drug is being excreted by the body between doses, how can we calculate the quantity of the drug remaining in the body at any particular time?

To be specific, let's suppose the drug is ampicillin (a common antibiotic) taken in 250 mg doses four times a day (that is, every six hours). It is known that at the end of six hours, about 4% of the drug is still in the body. What quantity of the drug is in the body right after the tenth tablet? The fortieth? The n-th?

Note:
The **sum of a finite geometric series** is given by

$$S_n = a + ax + ax^2 + \cdots + ax^{n-1} = \frac{a(1-x^n)}{1-x}, \quad \text{provided } x \neq 1.$$

For $|x| < 1$, the **sum of the infinite geometric series** is given by

$$S = a + ax + ax^2 + \cdots + ax^{n-1} + ax^n + \cdots = \frac{a}{1-x}.$$

1. For each of the following infinite geometric series, find several partial sums and the sum (if it exists).

(a) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

(b) $1 + 2 + 4 + 8 + \cdots$

(c) $6 - 2 + \frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \cdots$

2. People who save money often do so by putting some fixed amount aside regularly. To be specific, suppose \$1000 is deposited every year in a savings account earning 5% a year, compounded annually. What is the balance, B_n, in dollars, in the savings account right after the n^{th} deposit?

3. Find the sum of the series in the exercises below.

a. $3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \cdots + \frac{3}{2^{10}}$

b. $-2 + 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$

c. $\sum_{n=4}^{\infty} \left(\frac{1}{3}\right)^n$

d. $\sum_{n=4}^{20} \left(\frac{1}{3}\right)^n$

e. $1 + 3x + 9x^2 + 27x^3 + \cdots$