Math 267 – Test 1 Class Time:______________ **Name:** KEY

Read all directions carefully! Be neat, and box all answers. Points will be deducted for not following directions, sloppiness or lack of relevant work shown.

PLEASE NOTE: "Any student who accesses a phone or any internet-capable device during an exam for any reason automatically receives a score of zero on the exam. All such devices must be turned off and put away and made inaccessible during the exam."

Section I: Multiple Choice. Write your answers in the table provided. If you think the answer is "none of the above", write in E. (6 pts each)

(1) Which of the following are true:

~~I.~~ For any vectors **u** and **v**, $||\mathbf{u}+\mathbf{v}|| = ||\mathbf{u}|| + ||\mathbf{v}||$.

II. For any vectors **u** and **v**, $\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$

~~III.~~ For any vectors **u** and **v**, $\mathbf{u}\times\mathbf{v} = \mathbf{v}\times\mathbf{u}$.

IV. For any vectors **u** and **v**, $\mathbf{u} + (\mathbf{v}+\mathbf{w}) = (\mathbf{u}+\mathbf{v}) + \mathbf{w}$.

A. I and II **B.** II and IV **C.** II, III, and IV **D.** They all are. **E.** None of these

(2) Which of the following are true:

I. In R^3, if lines L_1 and L_2 are orthogonal to the same plane then they are parallel to each other.

II. In R^3, if lines L_1 and L_2 are parallel to a third line L_3 then they are parallel to each other.

~~III.~~ In R^3, two lines either intersect or they are parallel

IV. In R^3, two planes either intersect or are parallel.

A. I , II and IV. **B.** II and IV **C.** II, III, and IV **D.** They all are. **E.** None of these

3. Find the angle (in degrees) between the vectors $<1,2,3>$ and <2,-1,7>. (Round to two decimal places.)

A. 81.41° **B.** 40.20° **C.** 72.02° **D.** 90.00° **E.** None of these

4. Find an equation of the plane through (2,1,3) that is parallel to the plane $x - 9y + 13z = 5$

A. $2x + y + 3z = 37$
B. $x - 9y + 13z = 32$
C. $x - 9y + 13z = 15$
D. $x + 9y + 13z = 5$
E. None of these

$2 - 9\cdot 1 + 13\cdot 3 =$
$-7 + 39$
$= 32$

5. Which one of the following are parametric equations for the tangent line to the curve

$$x = t^3, y = t^2, z = t$$

at the point $(-1,1,-1)$?

A. $x = -1 + 3t, \quad y = 1 - 2t, \quad z = -1 + t$

~~B.~~ $x = -1 + 3t^2, \quad y = 1 + 2t, \quad z = -1 + t$

C. $x = -1 - 3t, \quad y = 1 + 2t, \quad z = 1 - t$

D. $x = -1 + 3t, \quad y = 1 - 2t, \quad z = 1 + t$

E. None of these

$\frac{dr}{dt} = \langle 3t^2, 2t, 1\rangle$
at $t = -1$
$v = \langle 3, -2, 1\rangle$

6. Which vector is in the same direction as $\langle 1,2,2\rangle$ and has length 6?

A. $\langle\frac{1}{3},\frac{2}{3},\frac{2}{3}\rangle$ **B.** $\langle\frac{2}{3},\frac{4}{3},\frac{4}{3}\rangle$ **C.** $\langle 2,4,4\rangle$ **D.** $\langle\frac{1}{6},\frac{2}{6},\frac{2}{6}\rangle$ **E.** None of these

$6\cdot\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\rangle = (2,4,4)$

7. A force $\mathbf{F} = \langle 3,3,1\rangle$ moves an object from the point (0,1,2) to the point (3,6,1). Find the work (ignoring units).

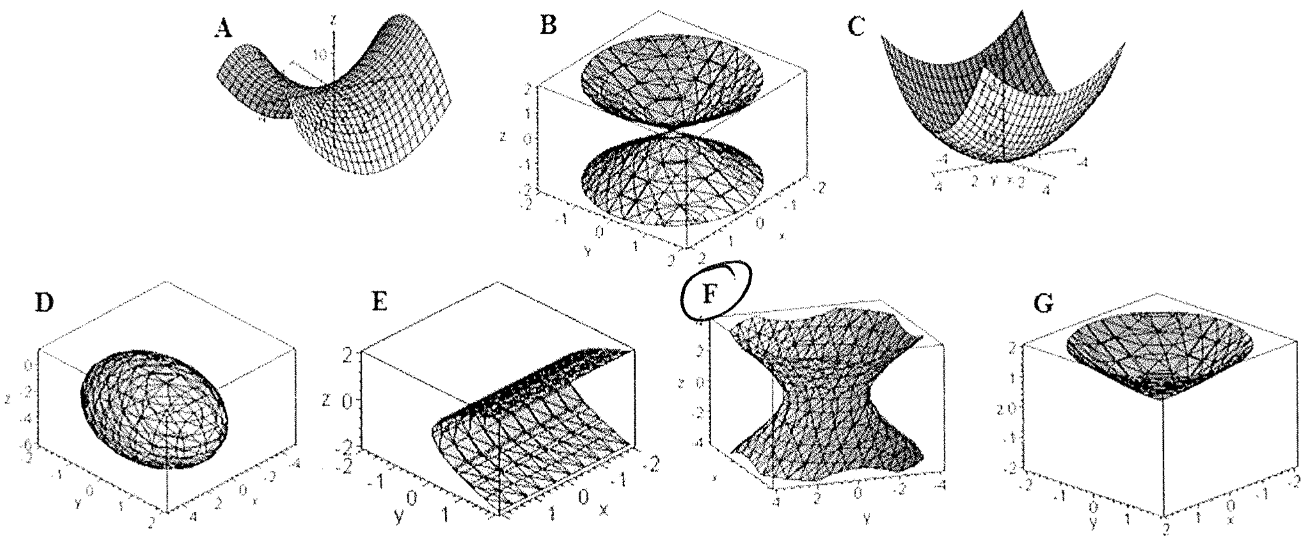
A. 19 **B.** 0 **C.** 21 **D.** 23 **E.** None of these

$D = \langle 3, 5, -1\rangle$
$F = \langle 3, 3, 1\rangle$

$F\cdot D = 9 + 15 - 1 = 23$

$x^2+y^2=3+z^2$

8. Which graph below corresponds to $x^2 + y^2 - z^2 = 3$?



9. Find the arc length of the curve:

$\mathbf{r}(t) = \langle (\sqrt{3})t, \sin t, \cos t \rangle$ where $0 \le t \le 3$

$\int_0^3 \sqrt{(\sqrt{3})^2 + (\cos t)^2 + (-\sin t)^2}\,dt$

$= \int_0^3 \sqrt{3+1}$

$= \int_0^3 2 \, dt$

A. 6 **B.** $3\sqrt{3}$ **C.** 12 **D.** $6\sqrt{3}$ **E.** None of these

10. Which of the following points is closest to the yz-plane?

A. (10,2,6) **B.** (−3,0,0) **C.** (2,7,100) **D.** (1,3,9)

Multiple Choice Table

Question	Answer
1	B
2	A
3	B
4	B
5	A
6	C
7	D
8	F
9	A
10	D

Section II: Free Response. Show all work and be neat.

(11) [10 pts] Determine whether the lines

$$L_1:\ x = 2 + t, \qquad y = 21 + 7t,\ \ z = 15 + 4t$$

$$L_2:\ x = -5 + 2t, \qquad y = -18 + 9t,\ \ z = -11 + 7t$$

intersect , are skew, or are parallel. If they intersect determine the point of intersection; if not leave that part of the question blank.

$$\begin{bmatrix} 2+t = -5+2s \\ 21+7t = -18+9s \\ 15+4t = -11+7s \end{bmatrix} \quad \begin{bmatrix} t-2s = -7 \\ 7t-9s = -39 \\ 4t-7s = -26 \end{bmatrix} \longrightarrow \begin{matrix} t = -3 \\ s = 2 \end{matrix}$$

$$(2-3,\ 21-21,\ 15-12) = (-1, 0, 3)$$

Circle One: Intersect or Skew or Parallel.

Point of intersection: $(-1, 0, 3)$

(12) Decompose the vector $\mathbf{u} = \langle 3, -13\rangle$ into two orthogonal vectors, one parallel to $\mathbf{v} = \langle 1,2\rangle$ and one orthogonal to $\mathbf{v}$. [10 pts]

$$\text{proj}_v u = \left(\frac{u\cdot v}{v\cdot v}\right)v = \frac{3\cdot 1 + (-13)\cdot 2}{1^2+2^2}\, v = \frac{3-26}{5}\, v = \frac{-23}{5}\, v$$

$$= \left\langle \frac{-23}{5}, \frac{-46}{5} \right\rangle$$

$\text{proj}_{\boldsymbol{v}}\boldsymbol{u} =$ $\left\langle \frac{-23}{5}, \frac{-46}{5} \right\rangle$

$\text{orth}_{\boldsymbol{v}}\boldsymbol{u} =$ $\left\langle \frac{38}{5}, \frac{-19}{5} \right\rangle$

$$\text{orth}_v u = u - \text{proj}_v u = \langle 3, -13\rangle - \left\langle -\frac{23}{5}, \frac{-46}{5} \right\rangle$$

$$\begin{matrix} -65 \\ 46 \end{matrix}$$

$$= \left\langle \frac{38}{5}, \frac{-19}{5} \right\rangle$$

(13) [10 pts] Find an equation of the plane through the points P(0,0,0), Q(0,1,0), R(1,2,3)

$\vec{PQ} = \langle 0,1,0 \rangle \qquad \vec{PR} = \langle 1,2,3 \rangle$

$n = \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 3\vec{i} + 0j - k \qquad P = (0,0,0)$

$3(x-0) + 0(y-0) - 1(z-0) = 0$

Answer: $3x - z = 0$

(14) [10 pts] Find the velocity and position vectors of a particle with acceleration $\boldsymbol{a}(t) = \langle 0,0,2 \rangle$ and initial conditions $\boldsymbol{v}(0) = \langle 1,-2,3 \rangle$ and $\boldsymbol{r}(0) = \langle 2,0,2 \rangle$.

$\vec{v}(t) = \langle c_1, c_2, 2t + c_3 \rangle, \quad v(0) = \langle 1, -2, 3 \rangle$

So

$\vec{v}(t) = \langle 1, -2, 2t+3 \rangle$

$\vec{r}(t) = \langle t + c_4, -2t + c_5, t^2 + 3t + c_6 \rangle$

$\boldsymbol{v}(t) =$ $\langle 1, -2, 2t+3 \rangle$

$\vec{r}(0) = \langle 2, 0, 2 \rangle$

So

$\boldsymbol{r}(t) =$ $\langle t+2, -2t, t^2+3t+2 \rangle$

$\vec{r}(t) = \langle t+2, -2t, t^2+3t+2 \rangle$