

MAT 211 Exam 3 Review Questions with Answers

Section 8.5 Conditional Probability and Independence

1. Suppose that $P(A \cap B) = 0.4$ and $P(B) = 0.9$. Find $P(A|B)$.
2. Suppose that $P(A|B) = 0.7$, $P(A|B') = 0.5$, $P(B) = 0.4$. Find $P(A)$.
3. Roll a fair die twice. Suppose A and B are two independent events such that A=rolling a 3 in the first time and B=rolling a 3 in the second time. Find $P(A \cap B)$.
4. Two distinguishable, fair dice are rolled (one red and one green). Let A represent the event that the red die is a 2 or 4. Let B represent the event that the sum of the two dice is 6. Find the conditional probability, $P(B|A)$ and express your answer as a fraction in lowest terms.
5. There is a 30% chance of rain today and an 80% chance of rain tomorrow. Assume that the event that it rains today is independent of the event that it rains tomorrow. What is the probability that there will be no rain today or tomorrow?

Section 8.6 Bayes Theorem

6. Use the information in question (2) to find $P(B|A)$.
[Hint: Use a Tree Diagram or Bayes' Theorem. (Round to three decimal places)]
7. Of the students living in the dormitories at a University, 58% live at the west hall, and the rest at the south tower. A sandwich shop randomly mails a coupon for a free sandwich to 26% of those at the west hall, and to 19% of those living at the south tower. A student living in a dormitory is randomly chosen. Find the probability that this student does not receive a coupon. (Round your answer to four decimal places.) [Hint: Use a tree diagram or total probability formula]
8. A professor insists that all senior physics majors take his notorious physics aptitude test. The test is so tough that anyone *not* going on to a career in physics has no hope of passing, whereas 40% of the seniors who do go on to a career in physics still fail the test. Further, 95% of all senior physics majors in fact go on to a career in physics. Assuming that you fail the test, what is the probability that you will not go on to a career in physics? (Round your answer to four decimal places.) [Hint: Use a Tree Diagram or Bayes' Theorem]

9. A motor insurance company insures drivers in age group A, B and C. 40% of the customers are in group A, 25% are in B, and 35% are in group C. The company's record shows that each year, 2% of customers in age group A, 1% in group B and 1.5% in group C made a claim. Given that a driver made a claim, what is the probability that the driver is from age group C? (Round your answer to four decimal places.) [Hint: Use a Tree Diagram or Bayes' Theorem]

Section 9.1 Discrete Random Variables

10. Give the probability distribution for the indicated random variable. Three fair coins are tossed, and X is the square of the number of heads showing. [Note: Enter your probabilities as fractions].

x				
$P(X = x)$				

11. Let X be the upper number when two dice are rolled, or the common number if doubles are rolled. (Example, a roll of 4-3 would be given a value of 4 while a roll of 5-5 would be given a value of 5) Fill in the corresponding probability distribution table.

x						
$P(X = x)$						

12. A random variable, X has the probability distribution table as shown.

x	2	4	6	8	10
$P(X = x)$	0.1	0.3			0.2

Using the table above and assuming that $P(X = 6) = P(X = 8)$.

- a. Find $P(2 < X < 10)$. b. Find $P(X \leq 6)$.

Section 9.2 Binomial Distribution

13. Let X be a binomial random variable with $n = 7$ and $p = 0.1$.
Compute the given probabilities. $P(X = 5)$ (Round your answer to five decimal places.)
[Note: Use *Binomial distribution formula* and then check your answer using technology]
14. If you roll a fair die 8 times, what is the probability of throwing at most two 6s? (Round your answer to three decimal places).
15. According to a study, the probability that a randomly selected teenager studied at least once during the week was only 0.52. Let X be the number of teenagers who studied at least once during the week. What is the probability that **at least 5** of the students in your study group of 10 have studied in the last week? [**i.e.** Find $P(X \geq 5)$].
16. Your manufacturing plant produces air bags, and it is known that 30% of them are defective. Six air bags are tested.
- Find the probability that exactly one of them is defective. (Round your answer to four decimal places)
 - Find the probability that at least two of them are defective. (Round your answer to four decimal places).
17. Assume that on a standardized test of 100 independent questions, a person has a probability of 80% of answering any particular question correctly. Find the probability of answering between 80 and 90 questions, inclusive. (Round your answer to four decimal places.)

Section 9.3 Measures of Central Tendency

18. After bowling 5 games in a bowling league, James has exactly 150 average. What does he have to bowl in his sixth game to boost his overall average to exactly 152?
19. Fifty darts are thrown at a dartboard. The probability of hitting a bull's-eye is 0.2. Let X be the number of bull's-eyes hit. Calculate the expected value of X , $E(X)$.
20. A random variable, X , has the probability distribution table as shown.

x	-2	-1	0	1	2
$P(X = x)$			0.4	0.1	0.1

Assume that $P(X = -2) = P(X = -1)$. Compute the expected value of X , $E(X)$.

21. A roulette wheel has the numbers 1 through 36, 0, and 00. A bet on four numbers pays 8 to 1 (that is, if you bet \$1 and one of the four numbers you bet comes up, you get back your \$1 plus another \$8). How much do you expect to win with a \$1 bet on four numbers? (Round your answer to the nearest cent)

Section 9.4 Measures of Dispersion

22. The following is a sample of unemployment rates (in percentage points) in the US sampled from the period 1990-2004.

4.2, 4.7, 5.4, 5.8, 4.9

Compute the sample mean, \bar{x} and standard deviation, s using the formula method. (Round your answers to one decimal place). [Note: You can only use the calculator method to check your answer].

23. Using question 22, find the following:

- In what range does the *empirical rule* predict that approximately 95% of the unemployment rates will fall?
- What is the actual percentage of the employment rates fall in the range predicted in part (a)
- Which gives the more accurate prediction of the percentage in part (b), *Chebyshev's rule* or the *Empirical rule*? [Give reason(s)].

24. Using the probability distribution table in questions 20, and assuming that $P(X = -2) = P(X = -1)$. Calculate **the standard deviation** rounded to two decimal places. Show all work as to how you performed your calculations as if you did not have a calculator. [Hint: First, find $E(X)$].

25. Calculate the expected value, the variance, and the standard deviation of the given random variable X . (Round all answers to two decimal places.)
 X is the number of red marbles that Suzan has in her hand after she selects five marbles from a bag containing five red marbles and two green ones.

Section P1/P2 Random Variable (Continuous) and Probability Distribution Function

26. Consider the function $f(x) = x$ on $[0,2]$. Does $f(x)$ admits a probability density function on the interval $[0,2]$?
27. Determine the value for k so that $f(x) = 5x$ is a probability density function on the interval $[0,k]$.
28. Find the value of k for which the given function is a probability density function.
 $f(x) = 8k$ on $[-2,2]$
29. The Grade point averages of the Gourmet society are *uniformly distributed* between 2.5 and 3.5.
- Find the probability distribution function, $f(x)$ for this scenario.
 - Using part (a), find the probability that a randomly chosen member of the society has a grade point average between 3 and 3.2.
30. Suppose a website is losing its membership continuously at a rate of 5% per year. Use a suitable density function to calculate the probability that a randomly chosen member will be lost in the next four years. (Round your answer to four decimal places.)

Section 9.5 Normal Distribution

31. Let X be a normal distribution with the given mean and standard deviation. Find the indicated probability. (Round your answer to four decimal places.)
 $\mu = 31, \sigma = 20$, find $P(30 \leq X \leq 43)$
32. Let Z be the standard normal distribution. Find the indicated probability.
 $P(-0.82 \leq Z \leq 1.2)$. (Round your answer to four decimal places.)

For questions 33-35, first find the corresponding z-values by hand, then you may use your calculator or a z-table to find your results. Clearly state the method you used and how you calculated your results if you used a calculator.

33. **SAT** test scores are normally distributed with a mean of 500 and standard deviation of 100. Find the probability that a randomly chosen test-taker will score below 450. (Round your answer to four decimal place).
34. Using the information in question 33, what is the probability that a random chosen test-taker will score between 400 to 550 (Round your answer to four decimal places)

35. Using the information in question 33, what is the probability that a random chosen test-taker will score above 600? (Round your answer to four decimal place).
36. Suppose X is a normal random variable with mean, $\mu=100$ and standard deviation, $\sigma=10$. Find a such that $P(a \leq X \leq 100) = 0.03$. (Round your answer to one decimal place).
[Clearly state the method you used and how you calculated the result if you used the calculator].

Section P3: Mean, Variance and Standard Deviation of a Continuous Random Variable

37. Consider the density function $f(x)=3x^2$ on the interval $[0,1]$. Find the expected value $E(X)$, the variance $Var(X)$ and the standard deviation $\sigma(X)$ for the density function and round your answers to four decimal places
[Clearly state the method you used and how you calculated your result if you used the calculator]
38. Find the median of the random variable with the probability density function given in question 37 round your answers to four decimal places.
39. Assuming that workers' salaries in your company are *uniformly distributed* between \$10,000 and \$60,000 per year, calculate the average salary in your company.
40. Your company's new series "Avocado Comedy Hour" has been a complete flop, with viewership continuously declining at a rate of 35% per month. How long will the average viewer continue to watch the show? (Round your answer to the nearest year.)

Answers

- | | | | |
|------------------|----------|-------------------|------------------|
| 1. $\frac{4}{9}$ | 2. 0.58 | 3. $\frac{1}{36}$ | 4. $\frac{1}{6}$ |
| 5. 0.14 | 6. 0.483 | 7. 0.7694 | 8. 0.1163 |
| 9. 0.3333 | | | |

10.

x	0	1	4	9
$P(X = x)$	1/8	3/8	3/8	1/8

11.

x	1	2	3	4	5	6
$P(X = x)$	1/36	1/12	5/36	7/36	1/4	11/36

12a. 0.7

12b. 0.6

13. 0.00017

14. 0.865

15. 0.6712

16a. 0.3025

16b. 0.5798

17. 0.5571

18. 162

19. 10

20. -0.3

21. -5 cents

22. $\bar{x}=5$

$s=0.6$

23a. [3.8, 6.2]

23b. 100%

23c. Empirical rule, since 2 standard deviations of the mean corresponds to 95%.

24. 1.19

25. 3.57, 0.34, 0.58

26. No, since $\int_0^2 f(x)dx = 2 \neq 1$

27. $k = \sqrt{\frac{2}{5}}$

28. $k = \frac{1}{32}$

29a. $f(x) = 1$

29b. 0.2

30. 0.1813

31. 0.2457

32. 0.6788

33. 0.3085

34. 0.5328

35. 0.1587

36. $a = 99.25$

37. $E(X) = \frac{3}{4}$

$Var(X) = 0.0375$

$\sigma(X) = 0.1936$

38. 0.7937

39. \$35,000

40. 3 years

GOOD LUCK!!